A Novel Generative Paradigm for Carnatic Rhythmic Composition

Jagdish A. Krishnaswamy[a] and Robert Wells[b]

Author information: [a]Independent scholar (email: jagdish.ak@gmail.com); [b]University of Mary Washington (email: rwells@umw.edu).

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Abstract: Mathematical structures are deeply embedded in the aesthetics of South Indian Carnatic music. *Kōrvai*s, which are rhythmic compositions performed in triplets in various parts of a concert, are the culmination of this mathematical aesthetic. Each *kōrvai* has two parts, the first called the *pūrvārdha* and the second called the *uttarārdha*, both based on mathematical structures that are aesthetically acceptable according to certain constraints. These structures have evolved traditionally and can accommodate most rhythmic requirements. However, the traditional methods are inadequate in addressing certain specialized constraints. Therefore, this article revisits rhythmic composition through conceptualization of newer rhythmic patterns and use of algorithmic approaches to determine the components of the *kōrvai.* Specifically, we systematically address the need for newer rhythmic compositional paradigms in Carnatic music by first introducing the fundamentals of Carnatic rhythmic compositional theory and practice via a collection of mathematical models. We subsequently highlight the limitations of traditional methods and outline the requirements of newer approaches, which will be exemplified through four new *kōrvai*s. We further exemplify, via video-recorded performances, how the proposed approach can be easily extended beyond percussion solos to Carnatic vocal presentations. The final section considers how these algorithmic approaches to Carnatic rhythmic composition could also be useful in other, non-Carnatic systems of music, with newly composed examples for illustration.

Keywords: Carnatic music; rhythmic composition; *tāḷa*; *kōrvai*; algorithm; *eḍuppu*; arithmetic progression; rhythmic offset

Introduction

IN South Indian “Carnatic” (classical) music, traditional constraints on rhythmic designs can inspire musical creativity, but can also produce compositional or improvisational limitations. Despite the wide range of available mathematical tools, the most common approaches to overcoming these limitations have been either through new rhythmic patterns derived using trial and error, or through relaxation of certain constraints. Thus, in this article, we develop a collection of novel, mathematically generated rhythmic designs that can provide fresh creative possibilities for Carnatic musicians, while also being of interest to music theorists and composers more broadly.

[2] To achieve these goals, the first section of this article (Introduction) outlines our project’s primary goals and methods. The second section (Theoretical Concepts) then provides an overview and mathematical formalization of beat cycles (*tāḷa*s) and rhythmic subdivisions, yielding quantitative tools useful not only for the current study, but for future Carnatic rhythm research. We also introduce the musical context and basic structure of South Indian classical percussive music. After these preliminaries, we argue for the need for a more advanced, algorithmically based approach to rhythmic composition by highlighting “mathematically difficult” rhythmic structural requirements that traditional methods are inadequate to address. Finally, we discuss the aesthetic requirements of any novel proposed approach.

[3] In the third section (A Deterministic Approach to Rhythmic Compositions), we formally introduce the new algorithmic approach to rhythmic composition using the special case of *rūpaka tāḷa*, a 3-beat rhythmic cycle with 4 subdivisions per beat. After indicating musical constraints in this *tāḷa* that render traditional, trial-and-error-based approaches inadequate, we show that our deterministic, linear-algebraic approach can provide a potentially infinite number of elegant solutions that simultaneously satisfy all aesthetic and rhythmic requirements.

[4] While section three represents a fundamental shift in the way rhythmic compositions are handled by Carnatic percussionists, vocalists, and non-percussive instrumentalists, section four (Compositional Applications Beyond Carnatic Music) considers possible applications to musics outside the Carnatic tradition. Specifically, we discuss the notion of imposing musical constraints as an artistic choice, which allows the current study to be applied to Western classical composition, jazz, and other styles. The section will conclude with two newly written compositional illustrations.

THEORETICAL CONCEPTS

Rhythmic Cycles, Divisions, and Subdivisions: Mathematical Representations for *Tāḷa, Āvartana, Naḍai,* and *Eḍuppu*

Mathematical Preliminaries: Rhythmic Cycles and Rhythmic Divisions

[5] Time is continuous, but most musical compositions articulate time in rhythmic patterns that repeat periodically. A pictorial representation of how time is hierarchically broken into periodic chunks in Carnatic music is shown in Figure 1.

[6] At the highest level, we refer to each periodically repeating chunk as a rhythmic cycle or an *āvartana*. Within each *āvartana* are a finite number of equally-spaced divisions called *akṣara*s or "beats," and within each beat are a finite number of subdivisions. The number of beats and their grouping within a cycle define what is known as a *tāḷa* (Nelson 2017). Specifically, the *tāḷa* in Carnatic music is a particular configuration of beat groupings, called *anga*s, that are mostly (with exceptions) demarcated by *kriyā*s (characteristic hand gestures). Note that the example of *tiṣra tripuṭa tāḷa* in Figure 1 consists of seven-beat cycles divided into *anga*s of lengths 3, 2, and 2, respectively, and each beat is marked by a specific *kriyā*.

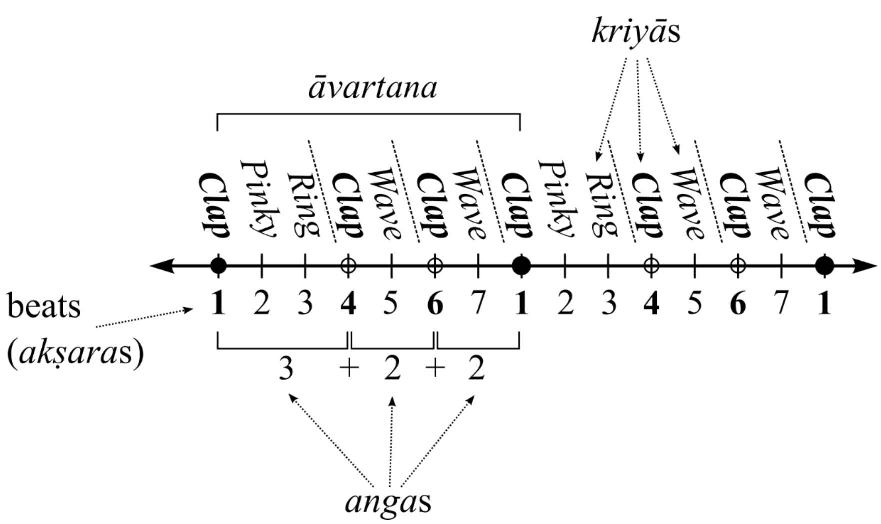


Figure 1: The internal structure of the Carnatic *tiṣra tripuṭa tāḷa,* which consists of seven-beat cycles divided into *anga*s of lengths 3, 2, and 2, respectively.

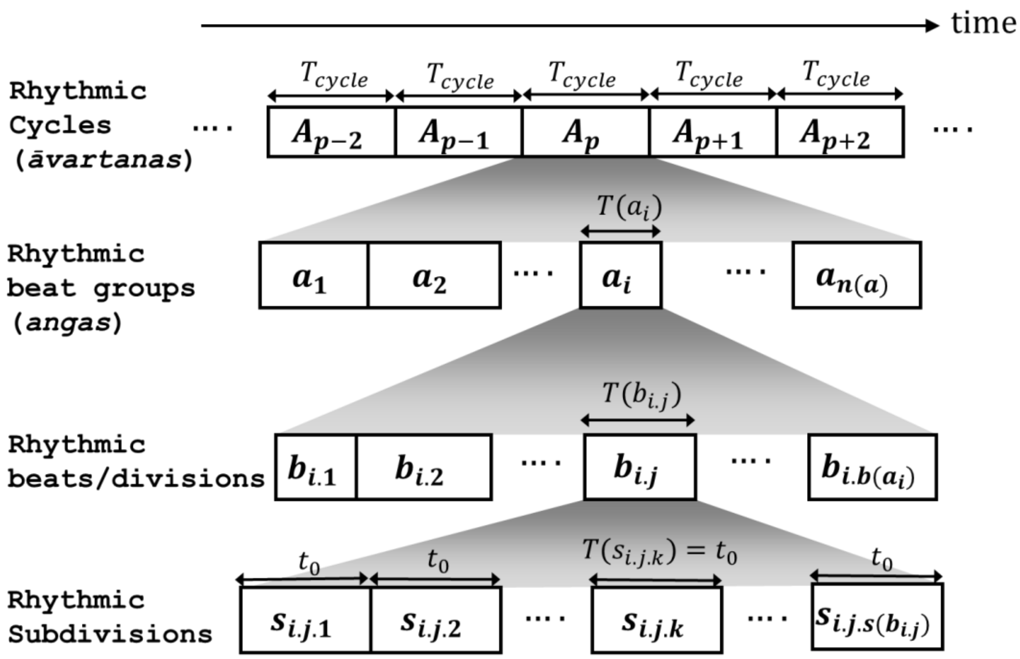


Figure 2: The three-level hierarchical structure of a rhythmic cycle (*āvartana*) of a *tāḷa*.

[7] We now present a broader, more mathematically based representation of Carnatic *tāḷa*; the reader uninterested in these mathematical details may skip to the next subsection (paragraph [16]). Figure 2 provides a generalized depiction of *tāḷa* structure and hierarchy.

[8] Here we denote an *āvartana* by A_p, where the index *p* refers to the position of the *āvartana* in time (A_1 is the first *āvartana* and A_p is the p^{th} *āvartana*). Within a given *tāḷa*, a single *āvartana* has duration T_{cycle} (measured in seconds) and consists of a finite number $n(a)$ of *anga*s, which are the largest divisions of a *tāḷa*. If each *anga* a_i has duration $T(a_i)$, then we have the equation given in Figure 3.

$$T_{cycle} = \sum_{i=1}^{n(a)} T(a_i)$$

Figure 3: Duration-based representation of an *āvartana* as the sum of its component *anga*s.

$$T_{cycle} = \sum_{i=1}^{n(a)} \sum_{j=1}^{b(a_i)} T(b_{i.j})$$

Figure 4: Duration-based representation of an *āvartana* as the sum of beats across all *anga*s.

[9] The i^{th} *anga* of the *tāḷa* has $b(a_i)$ beats/divisions. Each beat is physically displayed through a distinct hand movement (patting using the palm, the back of the hand, or the fingers) as the *tāḷa* proceeds. The j^{th} beat of the i^{th} *anga* (denoted by $b_{i.j}$) has duration $T(b_{i.j})$, yielding the equation in Figure 4.

[10] Finally, each beat consists of basic time units we will call "subdivisions." The j^{th} beat of the i^{th} *anga*, $b_{i.j}$, has $s(b_{i.j})$ subdivisions. The k^{th} subdivision of the j^{th} beat of the i^{th} *anga*, denoted by $s_{i.j.k}$, has duration $T(s_{i.j.k})$. The subdivisions of a beat are not represented by distinct hand movements. They simply fill time between beats and can be said to constitute *laya*. Therefore, the time interval T_{cycle} of one rhythmic cycle of the *tāḷa* can be expressed as shown in Figure 5.

[11] The formula in Figure 5 shows a generic 3-level hierarchical breakdown of an *āvartana* sequentially into *anga*s, beats, and subdivisions. Typically, the subdivisions are basic time units of equal duration, forming the most granular level of rhythmic structure.[1] Hence, $T(s_{i.j.k}) = t_0$ for all i, j, k, where t_0 is a constant, allowing simplification of the expression in Figure 5 to that in Figure 6.[2]

$$T_{cycle} = \sum_{i=1}^{n(a)} \sum_{j=1}^{b(a_i)} \sum_{k=1}^{s(b_{i.j})} T(s_{i.j.k})$$

Figure 5: Duration-based representation of an *āvartana* as a sum of subdivisions across all beats within all *anga*s.

$$T_{cycle} = \sum_{i=1}^{n(a)} \sum_{j=1}^{b(a_i)} s(b_{i.j}) t_0$$

Figure 6: Simplified representation for the duration of an *āvartana* when subdivisions have constant length t_0.

$$T_{cycle} = \sum_{i=1}^{n(a)} \sum_{j=1}^{b(a_i)} n_s t_0 = n_s t_0 \sum_{i=1}^{n(a)} b(a_i)$$

Figure 7: Simplified representation for the duration of an *āvartana* when each beat has n_s subdivisions.

[12] Furthermore, all beats of the *tāḷa* may have the same number of subdivisions, or some beats may have unequal numbers of subdivisions. The latter class of *tāḷa*s, whose beats vary in duration, are referred to as the *cāpu tāḷa*s.[3] Because *cāpu tāḷa*s and *tāḷa*s containing *cāpu* components need to be treated more rigorously for theoretical completion, we restrict our discussion in this paper to the subset of *tāḷa*s whose beats have equivalent numbers of subdivisions. The *cāpu tāḷa*s and their derivatives will be treated separately in future work. Therefore, for all *i* and *j*, $s(b_{i,j}) = n_s$, a constant. The Figure 6 equation then simplifies to the expression in Figure 7.

[13] It is possible for multiple *tāḷa*s with different *anga* structures to have the same number of beats, and hence the same number of total subdivisions in a cycle. For the purposes of creating rhythmic compositions (*kōrvai*s, as described in the following sections), these *tāḷa*s are essentially the same, because we are mainly concerned with the total number of subdivisions within an *āvartana* of the *tāḷa*. Therefore, we will simplify the above equation by removing the *anga*s from consideration and thinking only in terms of the number of beats and their subdivisions. We represent the total number of beats, across all the *anga*s of a *tāḷa*, using the parameter n_b, which is given by the summation $n_b = \sum_{i=1}^{n(a)} b(a_i)$. Figure 7 therefore reduces to Figure 8.

$$T_{cycle} = n_s t_0 \sum_{i=1}^{n(a)} b(a_i) = n_b n_s t_0$$

Figure 8: Simplified representation for the duration of an *āvartana* based only on the number of beats n_b, the number of subdivisions per beat (n_s), and the common length of these subdivisions (t_0).

[14] In other words, the total number of subdivisions in an *āvartana* of the *tāḷa* is $\frac{T_{cycle}}{t_0} = n_b n_s$, which is the product of the total number of beats per cycle and the number of subdivisions per beat. This product will be central to our calculations in later sections.

[15] The most familiar examples of *tāḷa*s having beats of equal durations are the members of the thirty-five *suḷādi sapta tāḷa*s, or "primordial seven talas"—a theoretical *tāḷa* scheme codified by composer and teacher Purandara Dasa (Nelson 2017). This group includes the common example of *ādi tāḷa*, which has 8 beats of equal duration. These beats are grouped into three *anga*s of 4, 2,

and 2 beats, respectively. Another example, especially important for this paper, is *rūpaka tāḷa*, containing 3 beats of equal duration.

Rhythmic Subdivisions: The Concepts of *Naḍai* and Speed

[16] The number of subdivisions within a beat is determined by two parameters—*naḍai* and speed (*kālam*). The *naḍai* defines how each beat of a *tāḷa* is broken down into subdivisions of equal duration (Chan 2013). In Carnatic music, *naḍai*s are given a name, and an integer n_{base} can be associated with each. We may also speak of various *speeds*, which refer not to contrasting tempos, but to changes in rhythmic density (e.g., via rhythmic augmentation or diminution) over a constant *tāḷa*. Namely, "first speed" corresponds to a melody or rhythm's basic form, while "second speed" is twice as rhythmically dense as first speed, "third speed" is twice as rhythmically dense as second speed, and so forth, all over constant *tāḷa*. We denote the speed parameter by an integer n_{speed}, where the first speed ($n_{speed} = 1$) of the *naḍai* involves n_{base} subdivisions per beat. The next speed, $n_{speed} = 2$, has double the number of subdivisions, that is $2n_{base}$ subdivisions per beat. Higher speeds would involve subsequent doubling.[4] Therefore, for a particular *naḍai*, the number of subdivisions is given by $\boldsymbol{n_s = n_{base} 2^{n_{speed}-1}}$.

[17] The number of subdivisions per beat, n_s, indicates the *naḍai* and speed and therefore, essentially, the internal structure of a *tāḷa*, while the beats (*akṣara*s) and *anga*s form a *tāḷa*'s external structure. Table 1 lists the commonly used *naḍai*s in Carnatic music and their n_{base} values, while Figure 9 graphically represents the various *naḍai*s.[5]

naḍai	$\boldsymbol{n_{base}}$
caturaṣra	1
tiṣra	3
khanḍa	5
miṣra	7
sankīrṇa	9
sampūrṇa khanḍa	10

Table 1: *Naḍai*s and their n_{base} values.

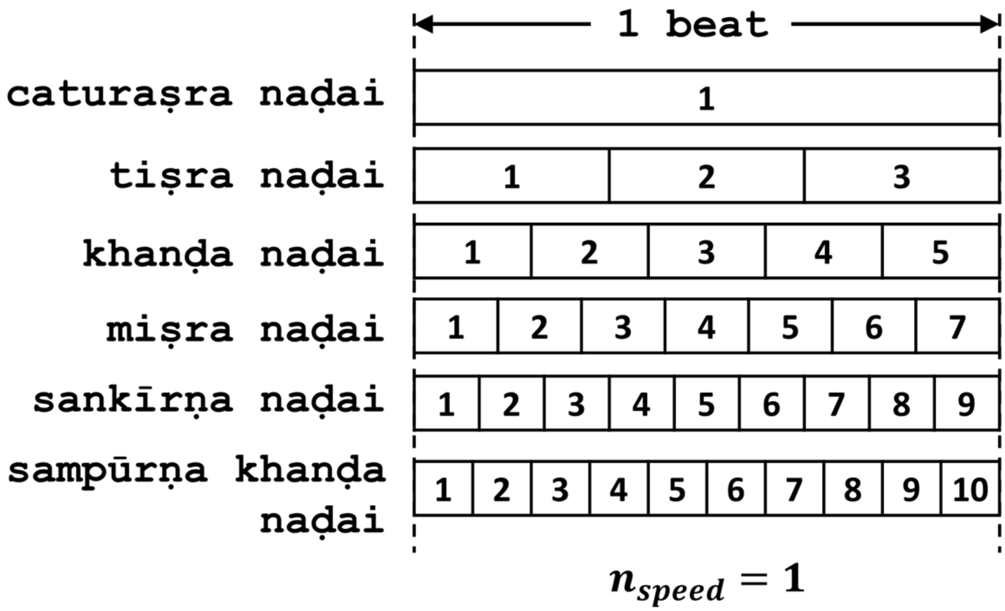


Figure 9: Visual model of the base speed (first speed, $n_{speed} = 1$) of various *naḍai*s in Carnatic rhythm.

[18] Theoretically, one could even expand the above set to include additional *naḍai*s. Although higher *naḍai*s might lack a traditional name, the n_{base} integer is sufficient to describe such *naḍai*s mathematically.

[19] From a practical perspective, we highlight that most compositions in South Indian Carnatic music are based on the *caturaṣra naḍai* (which corresponds to $n_{base} = 1$), for which $n_s = \mathbf{1} \times 2^{n_{speed}-1}$. When $n_{speed} = 1$ (first speed), we have one subdivision per beat, while $n_{speed} = 2$ (second speed) corresponds to two subdivisions per beat, $n_{speed} = 3$ (third speed) corresponds to four subdivisions per beat, and so on, as shown in Figure 10.

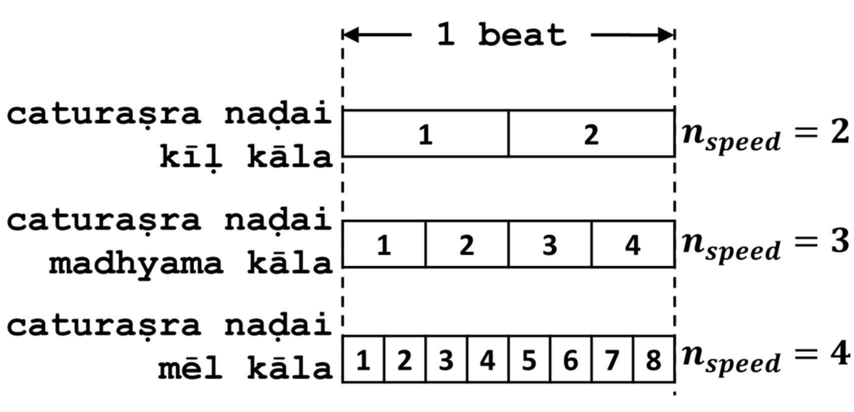


Figure 10: Visual model of speed variations in *caturaṣra naḍai* (*kīḷ kāla, madhyama kāla*, and *mēl kāla*).

[20] Traditionally, most reference calculations for rhythmic compositions are made with $n_{speed} = 3$, which is referred to as *madhyama kāla* ("intermediate speed") and has four subdivisions per beat. When $n_{speed} = 2$, corresponding to two subdivisions per beat, we have *kīḷ kāla* ("lower speed"), and when $n_{speed} = 4$, corresponding to eight subdivisions per beat, the designation is *mēl kāla* ("higher speed"). This said, the value of n_{speed} that defines *madhyama kāla* is not universally agreed upon, as these nomenclatures have evolved mostly informally. Therefore, from a formal theoretical viewpoint, it makes sense to view the concepts of *naḍai* and speed as numerical entities without getting entangled in naming conventions.

[21] Before applying these *naḍai* and speed formulations to specific *tāḷa*s, we should recognize that numerous *tāḷa*s, ranging from three beats to as many as 128 beats (the *simhanandana tāḷa*) per cycle, contain equal beat durations. These *tāḷa*s' widely varying *anga* structures necessitate taxonomic details that are beyond the scope of this article. As discussed earlier, specialized hand and finger motions (*kriya*s*)* are used to indicate divisions within each rhythmic cycle of these *tāḷa*s.

[22] For our rhythmic calculations, all *tāḷa*s with equal-length beats, regardless of *anga* structure, are adequately described by their number of beats, n_b, and the number of subdivisions within a beat, n_s. The number of subdivisions is further determined by the parameters n_{base} (*naḍai*) and n_{speed} (speed). To generalize the approach we are developing, we adopt the following notation to name a *tāḷa*. We denote a *tāḷa* using the letter C, referring to the fact that the *tāḷa* defines a rhythmic **C**ycle, and append n_b and n_s as subscripts, yielding the complete name C_{n_b,n_s}. The total number of subdivisions in each rhythmic cycle is then $n_b \times n_s$. For instance, for *ādi tāḷa* in *caturaṣra naḍai* ($n_{base} = 1$) and *madhyama kāla* ($n_{speed} = 3$), we would have eight beats ($n_b = 8$) divided into four subdivisions each ($n_s = n_{base}2^{n_{speed}-1} = 1 \times 2^{3-1} = 4$), yielding the cycle name $C_{8,4}$. Thus, the total number of subdivisions per cycle is $8 \times 4 = 32$.

The Role of Rhythmic Compositions in Indian Classical Music: The Concept of the *Kōrvai*

[23] To understand the role of rhythmic and percussive compositions in a Carnatic concert, we must understand the various parts of a musical presentation. Any composition presented in a concert has the following attributes: (a) *rāga*, which defines a composition's melodic framework and is marked by characteristic notes, groups of notes, and ornaments (*gamaka*s); and (b) *tāḷa*, which defines a composition's temporal framework. In addition to singing/playing a particular composition, the artists also exhibit other forms of musical creativity, such as performing precomposed passages of their own or improvising. A culmination of these creative aspects occurs in the "main item" of a concert, a composition we will call the "reference composition." Typically, toward the midpoint of a full concert, which usually lasts for 2.5–3 hours, the leading artist begins the main presentation centered around the reference composition. Table 2 provides the details of the sequence of presentations that occur in the "main item."

Section No.	Presentation	Description
1	*Ālāpana*	Creative melodic embellishment of the *raga* that is not tied to a *tāḷa* (Kassebaum 1987).
2	*Tānam*	Improvisatory form incorporating short phrases that are loosely tied to some rhythmic framework (Ravikiran 2007).
3	Reference composition	Precomposed composition/song (usually a *kruthi* or a *rāgam-tānam-pallavi*) around which the main presentation is centered; generally constructed using one *rāga* and *tāḷa*; music expresses the song's meaning.
4	*Niraval*	The main artist creatively embellishes a meaningful verse from the reference composition (Radhakrishnan 2012). The point in the rhythmic cycle (*āvartana*) where the reference verse begins, called the *eḍuppu*, may start on the first subdivision of the cycle, or may be shifted by a few subdivisions with respect to the start of the *āvartana*.
5	*Kalpanaswara/ Swarakalpana*	Spontaneous presentation of notes typically terminating at the start of the reference verse on which the *niraval* was performed (Ranjani and Sreenivas 2013). Typically concludes with major rhythmic structures such as the *koraippu* (a set of melodic patterns that sequentially decrease in length) and a group of three *kōrvai*s, sub-compositions that mark the end of the *swarakalpana* section and transition back to the reference verse. The third *kōrvai* should end exactly at the *eḍuppu*, where the reference verse starts.
6	*Tani āvartana*	Percussion solo performances after main artist and melodic accompanists have finished their creative presentations around the main item (Nelson 1991). Consists of free, alternating patterns followed by a trio of *kōrvai*s terminating at the *eḍuppu*. After multiple rounds of alternating presentations by the percussion artists, often involving explorations of different *naḍai*s, the percussion artists perform a *koraippu* and finish with a set of three *kōrvai*s. Again, the third *kōrvai* terminates at the *eḍuppu*, after which the main artist picks up the reference verse and finishes presenting the main item of the concert.

Table 2: A brief summary of a typical "main item" in a Carnatic concert. This main item could be in the form of melodic and percussive creative decorations around a reference composition, or the more detailed case of a *rāgam-tānam-pallavi* (beyond the scope of this paper).

[24] The last two items of the above table—the *swarakalpana* and the *tani āvartana*—involve the concept of *kōrvai,* which is the most important rhythmic compositional concept in Carnatic music. An artist's quality is often measured through their ability to compose *kōrvai*s that present rich mathematical ideas in easily understandable, elegant forms, such that the presentation is intriguing to the expert and enjoyable to the lay audience. *Kōrvai* composition entails some important rhythmic constraints that we consider in paragraphs [26]–[29].

The Concept of Rhythmic Offset or *Eḍuppu*: *Atīta* and *Anāgata*

[25] Before exploring *kōrvai*s in greater detail, we need a precise, quantitative characterization of *eḍuppu*. As indicated in Table 2, the *eḍuppu* represents the number of subdivisions from the start of a rhythmic cycle (*āvartana*) that the reference verse of a composition begins. This reference verse can begin at the start of a rhythmic cycle, but also before or after. In Carnatic music, if the verse begins before the start of the rhythmic cycle, it is called an *atīta eḍuppu*, and if it begins after, it is called an *anāgata eḍuppu*. We indicate an *anāgata eḍuppu* of n_{shift} subdivisions by a positive integer $+n_{shift}$ and an *atīta eḍuppu* of n_{shift} subdivisions by a negative integer $-n_{shift}$.

Current Approaches and Challenges in Carnatic Rhythmic Composition

Traditional *Kōrvai* Models and Constraints

[26] *Kōrvai*s are presented in threes, as discussed in the previous section, thus forming sub-compositions of a larger tripartite rhythmic composition.[6] Traditionally, the *kōrvai* trio satisfies one of the following two models (Nelson 2008):

Model A: The *kōrvai* lengths are identical, with each sub-composition having the same total number of rhythmic subdivisions. Specifically, if each sub-composition has R (for "**R**hythm") subdivisions, the complete rhythmic composition has $3R$ subdivisions.

Model B: The *kōrvai* lengths are structured in arithmetic progression, where the three sub-compositions have $R - r, R, R + r$ subdivisions, respectively (where r is a positive integer), again summing to $3R$ subdivisions overall. In the process, the parts of a *kōrvai* undergo arithmetically progressive modifications, as we will see in the following subsections.

[27] Conventional performance practice dictates certain constraints in the creation of rhythmic compositions.[7] These are:

Constraint 1: The rhythmic composition must end at the exact point in the rhythmic cycle (the *eḍuppu*, described by the integer n_{shift}) where the reference verse of the main composition starts, so that the reference composition can start immediately after its conclusion.

Constraint 2: For cognitive simplicity, only patterns involving arithmetic numerical progressions (e.g., 1, 2, 3, 4… or 2, 5, 8, 11…, where pattern lengths increase/decrease by a constant value) should be used. (Non-arithmetic progressions such as 1, 2, 4, 8… or 0, 1, 1, 2, 3, 5… are less understandable to the listener and hence not preferred). These could be arithmetic progressions in a local context (within a single *kōrvai*) or in a global context (across the three *kōrvai*s).

Constraint 3: The start of the rhythmic composition should coincide with the start of a rhythmic cycle.

Constraint 4: All subdivisions should be the same duration. This means that we are constrained to maintain the same *naḍai* (n_{base}) and speed (n_{speed}) throughout the rhythmic composition.

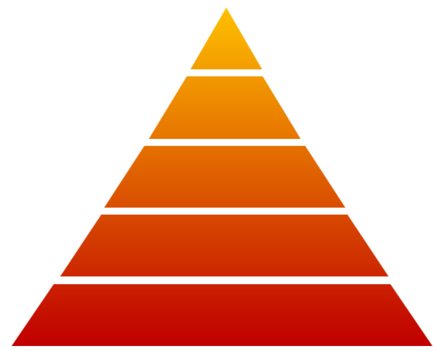
[28] Performers do not typically compromise on constraints 1 and 2. While there are cases where *kōrvai*s can be played in different *naḍai*s (in violation of constraint 4), the current project will respect constraint 4.[8] Constraint 3 is routinely compromised in many cases due to limitations with the current compositional approaches, as will be explained in the next section.[9] In fact, it is quite common for an artist to respect this constraint on some occasions and neglect it on others. Generally, though, the customary practice is to start a rhythmic composition (three sub-compositions) on the first subdivision of a rhythmic cycle, and only begin at a shifted position n_{shift} (in violation of constraint 3) if necessitated by the *eḍuppu.*

[29] The current study, however, seeks to identify situations where it is mathematically impossible to obey constraint 3 while satisfying the other three constraints. We will remedy such situations by conceptualizing new patterns, thereby pushing the limits of rhythmic composition. The choice of respecting constraints, we believe, lies in the hands of the artist, and the subsequent execution represents the artist's individuality. Although only constraints 1 and 2 are mandatory, we believe that imposing additional constraints, such as constraints 3 and 4, challenges an artist or composer to craft novel generative approaches to rhythmic composition. Such approaches could reflect traditional aesthetics while, ideally, being enjoyably innovative and intellectually stimulating. Another argument for the imposition of additional "creative" constraints, such as constraints 3 and 4, is that the new resulting rhythmic structures can easily be generalized to broader musical situations, including cases where constraints 1–4 are not uniformly imposed.

Traditional Internal Structure of a Rhythmic Sub-Composition or *Kōrvai*

[30] Before we explore more advanced conceptions of the *kōrvai*, including new compositional methods and patterns, a clear understanding of traditional *kōrvai* construction is necessary. Traditionally, the three sub-compositions that make up the rhythmic composition have an internal two-part structure. The first part is an introductory section called the *pūrvārdha*. The second part is a concluding section called the *uttarārdha*. Tables 3 and 4 summarize common

structural paradigms used for the introductory and concluding parts of a sub-composition. In addition to text descriptions, we have provided illustrations to help the reader visualize the rhythmic patterns through shapes. Further, we emphasize that this is not an exhaustive list, but merely attempts to capture the basic elements of *kōrvai* composition as widely practiced. Many derivative structures are possible and have been implemented by numerous percussionists. Table 3 provides a semi-exhaustive list of typical *pūrvārdha* patterns, while Table 4 does the same for common *uttarārdha* patterns.[10]

Design #	Introductory part *(pūrvārdha)* design	Graphic visualization
P_1	This design consists of multiple sequential lines that are growing in arithmetic progression. Each consecutive line adds a set number of subdivisions. This is referred to as the *ṣrōtōvāha yati.*	
P_2	This design consists of multiple sequential lines that are shrinking in arithmetic progression. Each consecutive line subtracts a set number of subdivisions. This is referred to as the *gōpuccha yati.*	

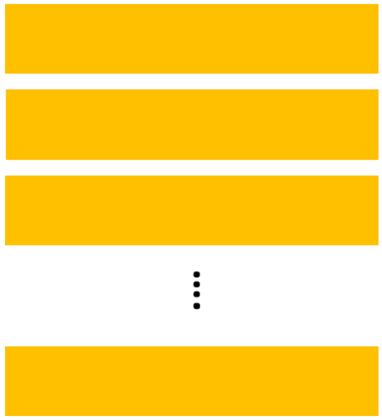
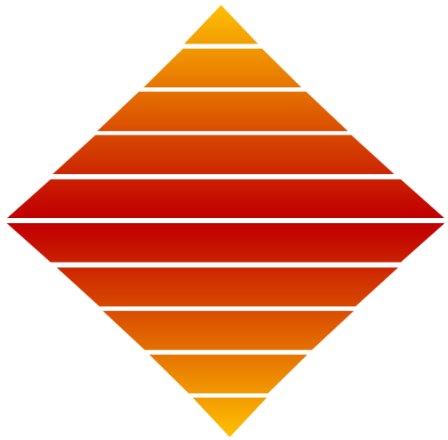
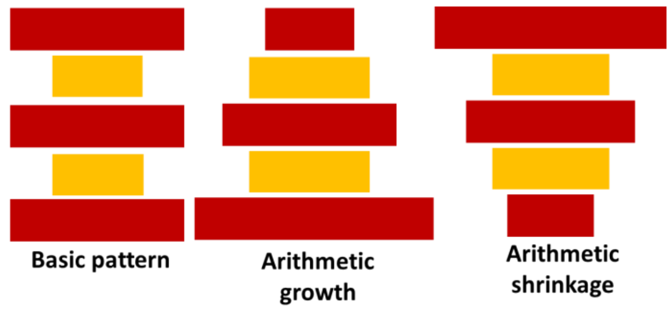
P_3	In this design, referred to as the *sama yati*, the same rhythmic pattern is played multiple times (at least thrice).	
P_4	This design consists of an arithmetically growing sequence of lines followed by an arithmetically shrinking sequence of lines. Because it resembles the shape of a *mṛdanga* drum, it is referred to as *mṛdanga yati*.	
P_5	This pattern consists of an arithmetically shrinking sequence of lines followed by an arithmetically growing sequence of lines. Owing to its resemblance to a *ḍhamaru* drum's shape, this pattern is referred to as *ḍhamaru yati*.	

Table 3: A semi-exhaustive summary of *pūrvārdha* patterns in the traditional Carnatic percussive context.

Design #	**Concluding part *(uttarārdha)* design**	**Graphic visualization**
U_1	This design consists of three rhythmic phrases, shown in red in the illustrations, separated by two equal spacers called *kārvai*s. One can introduce an arithmetic progression (growth or reduction) into the three rhythmic phrases, but not in the *kārvai*s.	
U_2	This pattern is a nested version of U_1. Specifically, the three phrases separated by *kārvai*s can themselves have a U_1 structure. As in the case of U_1, one can introduce an arithmetic progression over the three lines shown in light and dark green in the illustrations (and not over the *kārvai*s shown in yellow).	

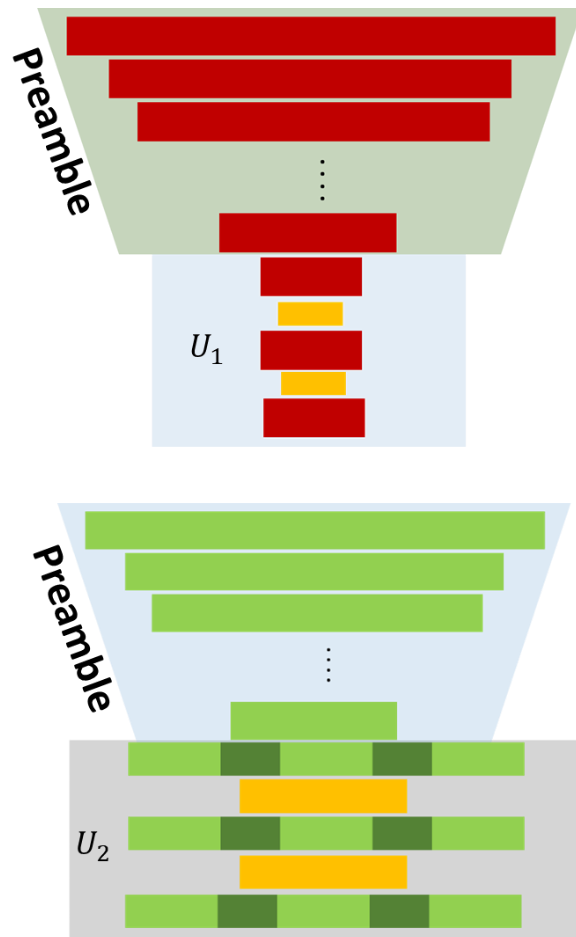
U_3	This is a specialized structure where a U_1 or U_2 pattern is preceded by a funneling preamble. The first line of the preamble is the first phrase of U_1 (or U_2) stretched by an integer factor n. The second line then stretches the opening phrase of U_1 by a factor $n-1$, and so on, in sequential descent until the first phrase of U_1 or U_2 appears. The last line of the funnel stretches the first phrase of U_1 (or U_2) by a factor 2.	

Table 4: A semi-exhaustive summary of *uttarārdha* patterns in the traditional Carnatic percussive context.

[31] We see from Tables 3 and 4 that arithmetic progressions can be introduced within the *pūrvārdha* and *uttarārdha* of a *kōrvai*. One can also apply an arithmetic progression to any of the rhythmic phrases constituting the *pūrvārdha* or *uttarārdha* across the *kōrvai* trio, in line with Model B described earlier in this section. Detailed mathematical representations of these *pūrvārdha* and *uttarārdha* patterns are provided in Appendix A1.

Challenges: The Case of the C_{n_b,n_s} Rhythmic Cycles where $n_b n_s = 0\ (mod\ 3)$

[32] Tables 3 and 4 demonstrate that in traditional approaches to rhythmic composition, if we assume constant *naḍai*, then the total number of rhythmic subdivisions across 3 *kōrvai*s will always be a multiple of 3. This is because, irrespective of the *tāḷa* and the shapes chosen for the *pūrvārdha* and the *uttarārdha*, the sub-composition will be repeated thrice as part of the *kōrvai*

trio. While not always a limitation, this property does make it impossible to design compositions in certain cases.

[33] For example, let us consider the class of rhythmic cycles C_{n_b,n_s} where the product $n_b n_s$ is a multiple of 3, so that each rhythmic cycle has $n_b n_s = 0\ (mod\ 3)$ subdivisions. This means that the number of beats in the *tāḷa* and/or the number of subdivisions per beat is a multiple of 3. Now, if the reference composition starts n_{shift} subdivisions before or after the first subdivision of the rhythmic cycle, where n_{shift} is *not* a multiple of 3, then it is impossible to design a rhythmic composition using traditional approaches (i.e., Model A or B from paragraph [26]) that simultaneously satisfies all the constraints listed in previous subsections. This is because if we begin the composition at the start of the rhythmic cycle and end it n_{shift} subdivisions before/after the start of another cycle, then the total number of subdivisions in the composition must be $N_{total} = K(n_b n_s) + n_{shift}$ (where K is a positive integer), which is not a multiple of 3.[11] Specifically, the number of subdivisions spanned by a *kōrvai* trio would be equivalent to $n_{shift} \neq 0\ (mod\ 3)$, contradicting the aforementioned fact that a *kōrvai* trio's length is always equivalent to $0\ (mod\ 3)$ subdivisions. Therefore, the simultaneous satisfaction of constraint 3 and usage of traditionally derived *kōrvai* structures is impossible.[12]

[34] An example of such a "problem" *tāḷa* is $C_{3,4}$—*rūpaka tāḷa* in *caturaṣra naḍai (madhyama kāla)*—which has 12 subdivisions per cycle.[13] In many musical compositions set to the $C_{3,4}$ cycle, the reference composition or phrase starts 2 subdivisions before or after the start of the rhythmic cycle. In this case, $N_{total} = 12K \pm 2$ is not a multiple of 3 and hence would pose problems.[14] We emphasize that while the number of beats is a multiple of 3 here, similar problems can arise in *tāḷa*s having any number of beats, as long as the total number of subdivisions in a cycle is a multiple of 3 (e.g., *Ādi tāḷa* in *tiṣra naḍai* or $C_{8,3}$, totalling 24 subdivisions per cycle, or *miṣra cāpu tāḷa* in *tiṣra naḍai*, totalling 21 subdivisions per cycle). While beyond the scope of this paper, problems satisfying the four aforementioned constraints could also arise in single- or multi-*naḍai* paradigms in which the number of subdivisions per *tāḷa* cycle is *not* a multiple of 3.[15]

[35] Conventional approaches tend to simplify the situation by relaxing constraint 3 and shifting the start of the rhythmic composition by n_{shift} subdivisions, parallel to the reference phrase with respect to the *tāḷa*. One can, then, still perform a composition whose *tāḷa* subdivisions form a multiple of 3 because the offset of n_{shift} (here, ± 2 subdivisions) has been counteracted. For example, consider the $C_{3,4}$ rhythmic cycle where we have $n_{shift} = +2$ (an *anāgata eḍuppu* of two subdivisions), and suppose a *kōrvai* measures 36 subdivisions, occupying 3 *āvartana*s of the *tāḷa*. One can then shift the *kōrvai* to start at the relevant *eḍuppu* (the third subdivision of the first beat). Since the starting point has been shifted forward by 2 subdivisions, the corresponding ending point is also shifted by $n_{shift} = +2$ subdivisions, allowing the rhythmic composition to conclude on the required *eḍuppu*. Figure 11 illustrates this with a 36-subdivision-long *kōrvai*

whose *pūrvārdha* measures 21 subdivisions. This *pūrvārdha* is made up of three equal parts, each lasting 7 subdivisions (*pūrvārdha* design P_3 [see Table 3]). The *uttarārdha* measures 15 subdivisions with three equal 5-subdivision parts (*uttarārdha* design U_1 where $x_2 = 0$ [see Table 4]).

Cycle#	Beat 1				Beat 2				Beat 3			
1			tha	ka	dhi	na	dhim	.	.	tha	ka	dhi
2	na	dhim	.	.	tha	ka	dhi	na	dhim	.	.	tha
3	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi	gi
4	na	thom	tha	ka	dhi	na	dhim	.	.	tha	ka	dhi
5	na	dhim	.	.	tha	ka	dhi	na	dhim	.	.	tha
6	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi	gi
7	na	thom	tha	ka	dhi	na	dhim	.	.	tha	ka	dhi
8	na	dhim	.	.	tha	ka	dhi	na	dhim	.	.	tha
9	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi	gi
10	na	thom										

Figure 11: An example of a traditional *kōrvai* design (Model A—see paragraph [26]) being used to compose a rhythmic trio for *rūpaka tāḷa* in the *caturaṣra naḍai* ($C_{3,4}$) with an *eḍuppu* of $n_{shift} = +2$. The *pūrvārdha* is shaded yellow and the *uttarārdha* blue, for clarity. The start of the *kōrvai*-trio has been shifted by +2 subdivisions corresponding to the *eḍuppu*.

[36] While this aesthetically successful approach has worked well for many years, efforts to identify new compositional approaches to help overcome this constraint have been limited. One set of solutions for this problem takes the form of what artist Chitravina Ravikiran calls "*keyless kōrvai*s" (Ravikiran 2015). However, Ravikiran admits to having "literally stumbled upon" most of the solutions (presumably via trial and error), making the approach difficult to codify. Moreover, while some of his new *kōrvai*s are easily comprehensible for both lay and expert listeners, other *kōrvai*s' complexity would create a significant obstacle for non-expert listeners. These considerations suggest the need for algorithmic intervention to codify new rhythmic structures that adhere closely to traditional aesthetics while pushing compositional boundaries. Such a deterministic, semi-algorithmic approach would allow us to move beyond mere trial and error. Moreover, incorporating all four aforementioned constraints, including constraint 3, when generating new rhythmic designs would lead to musically significant compositions that appeal to lay listeners and experts alike.

[37] Algorithmic musical compositions have been a popular topic of research in Carnatic music (Moroni et al. 2000).[16] However, these algorithms are not straightforward, and they involve musical aspects that vary between composers, performers, audiences, and social contexts. Algorithmic designs that involve training a machine to mimic the human brain (Chan, Potter, and Schubert 2006) offer one solution to the problems of variability and complexity. Automated music generation in Carnatic music carries the added challenge of having to encompass complex aspects such as *gamaka*s (melodic ornamentation) and idiomatic rhythmic patterns (Garani and Seshadri 2019). This said, there are ample possibilities in the percussion realm for machine-

generated music, the exploration of which is currently in preliminary stages (Trochidis, Guedes, and Anantapadmanabhan 2017; Trochidis et al. 2016). There is, therefore, prior precedent for our algorithmic development of newer patterns that can overcome the aforementioned issues while satisfying the constraints given in paragraph [27].

Aesthetic Requirements of Novel Approaches

[38] We have identified a need to develop newer deterministic strategies to compose rhythmic pieces bound by the four aesthetic constraints listed above. The algorithmic approach should facilitate designing new patterns that have progressive behavior, but whose total subdivisions do not sum to a multiple of three. A simple arithmetic-progression-based rhythmic structuring, either locally or in a global sense (as in Model B from paragraph [26]) with respect to a *kōrvai,* is inadequate. Thus, in the next section, we conceptualize patterns whose aesthetic basis is still the arithmetic progression, but whose internal structure breaks the multiplicity with three. Further, for various traditional and aesthetic reasons, it is important for the *pūrvārdha* and the *uttarārdha* to consist of a minimum of three rhythmic phrases (equal, sequentially growing, or sequentially shrinking in size).[17] This will ensure that the newly developed patterns are backward-compatible with the schemes that have evolved over years of percussive practice. We impose the above 3-subpattern minimum as an additional requirement beyond the four previously discussed constraints of *kōrvai* composition.

A Deterministic Approach to Rhythmic Compositions

New Progressive Patterns: Core Design Principle

[39] As previously indicated, the problem with the current approach to rhythmic compositions is that the total number of subdivisions in a *kōrvai* trio is always a multiple of three. This would not allow us to simultaneously satisfy all constraints laid out paragraph [27] for the C_{n_b,n_s} $(n_b n_s = 0\ (mod\ 3))$ family of *tāḷa*s having *eḍuppu*s $n_{shift} \neq 0\ (mod\ 3)$. In designing new patterns, then, we must use arithmetically progressive patterns that break the multiplicity with three. We achieve this by introducing cumulative progressions rather than simple arithmetic progressions. Namely, each of our novel rhythmic designs will contain a component that steadily grows/shrinks across three *kōrvai*s. This progressive growth/shrinking maintains an arithmetic pattern while breaking the multiplicity with 3. In the following subsections, we describe how to construct these new compositional designs.

Proof-Of-Concept Examples

[40] We demonstrate our novel compositional strategy using two rhythmic designs that introduce a non-multiple-of-three component into the *kōrvai* trio. In the first design, this new component is applied to the *pūrvārdha* of the *kōrvai* and in the second, to the *uttarārdha*. We illustrate the two

designs using the $C_{3,4}$ rhythmic cycle with $n_{shift} = +2$ (*anāgata eḍuppu* of two subdivisions) and $n_{shift} = -2$ (*atīta eḍuppu* of two subdivisions), respectively. These two cases correspond to reference compositions starting on the third and eleventh subdivisions, respectively, of the $C_{3,4}$ cycle measuring twelve subdivisions. We choose these *eḍuppu*s as illustrative examples because of their use in numerous Carnatic compositions. This said, the rhythmic conceptualizations and algorithmic approach developed here are applicable to all periodic rhythmic constructs, including the *cāpu tāḷa*s.

Design 1: Cumulatively Progressive *Pūrvārdha*s

[41] Figure 12 shows the proposed structure of the *pūrvārdha*s and the *uttarārdha*s of the three *kōrvai*s with the aforementioned cumulatively progressive component applied in the *pūrvārdha*. In the figure, the melodic/rhythmic units forming the *pūrvārdha*s (described below) have lengths a_1 and a_2, while those forming the *uttarārdha*s have lengths x_1 and x_2, respectively. Table 5 summarizes the resulting subdivision counts for each internal pattern. For context, recall that in a more traditional *kōrvai* trio, the sequence of *pūrvārdha*s would have been $(3a_1 - r_p, 3a_1, 3a_1 + r_p)$ for some integer $r_p \geq 0$.

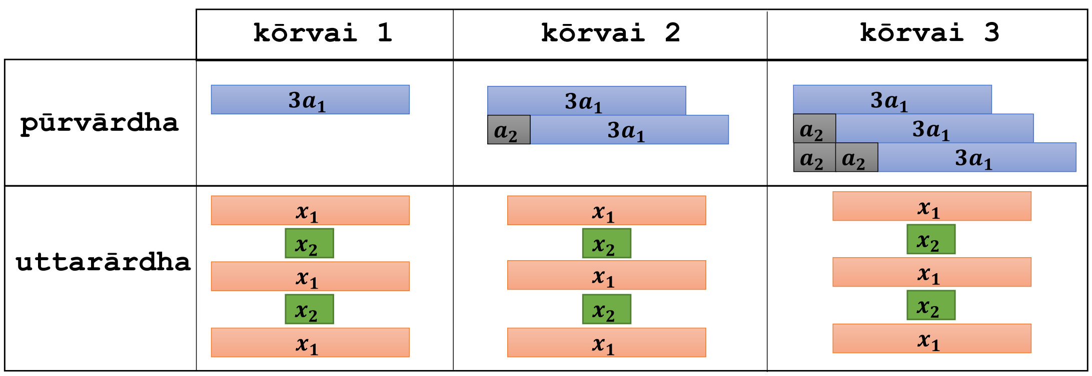


Figure 12: A geometric representation of the *pūrvārdha* and *uttarārdha* of the three *kōrvai*s with the first newly proposed rhythmic composition structure (“Design 1”).

Kōrvai #	*Pūrvārdha* pattern (in rhythmic subdivision counts)	*Uttarārdha* pattern (in rhythmic subdivision counts)
1	$3a_1$	$3x_1 + 2x_2$
2	$(3a_1, 3a_1 + a_2)$	$3x_1 + 2x_2$
3	$(3a_1, 3a_1 + a_2, 3a_1 + 2a_2)$	$3x_1 + 2x_2$
Total subdivisions	$18a_1 + 4a_2$	$3(3x_1 + 2x_2)$

Table 5: Subdivision counts for the *pūrvārdha* and *uttarārdha* of the 3 *kōrvai*s forming the Design 1 rhythmic composition trio.

[42] The base pattern for the introductory section is a trio of identical phrases having a_1 subdivisions each, yielding $3a_1$ subdivisions total. When played just once, the pattern is aesthetically complete (resembling *pūrvārdha* design P_3) due to the triple repetition, while also allowing cumulative growth in the subsequent *kōrvai*s. This base pattern is the *pūrvārdha* of the first *kōrvai* of the series. The subsequent *kōrvai*s incorporate added phrases between statements of the $3a_1$ base pattern. In the second *kōrvai*, two $3a_1$ base patterns are separated by a phrase of length a_2, while in the third *kōrvai*, three base patterns are separated by phrases of length a_2 and $2a_2$ subdivisions, respectively. These additional phrases will be the crux of the new compositional designs because they break the multiplicity with three. The *uttarārdha* is the same in all three sub-compositions, consisting of Design U_1 ($3x_1 + 2x_2$ subdivisions).

[43] In this example, the total number of subdivisions of the three *uttarārdha*s together is a multiple of 3. However, the total number of subdivisions across the three *pūrvārdha*s is not necessarily a multiple of 3. This is because of the introduction of the progressively increasing component a_2, which contributes $4a_2$ total subdivisions across the three *kōrvai*s. With these added components, the rhythmic composition now satisfies all previously mentioned constraints (constraints 1–4) and the aesthetic requirements laid out in paragraph [38]. We have a progressively growing (or shrinking) *pūrvārdha* as we move through the three sub-compositions, while maintaining the same *uttarārdha*. Furthermore, we have used only arithmetically progressive patterns.

[44] The total number of subdivisions in the rhythmic composition will then be $18a_1 + 4a_2 + 9x_1 + 6x_2$. The constraint equation that needs to be satisfied, for a rhythmic cycle C_{n_b,n_s} with reference *eḍuppu* n_{shift}, is $\boldsymbol{18a_1 + 4a_2 + 9x_1 + 6x_2 = (n_b n_s)N_{cycles} \pm n_{shift}}$.

[45] The right side of the equation presents an alternative formula for the number of subdivisions in the rhythmic composition, where $n_b n_s$ represents the number of subdivisions in a single *āvartana*, and N_{cycles} is the number of complete *tāḷa* cycles contained in the composition. The right side thus represents the total number of subdivisions as N_{cycles} complete cycles (for some whole number N_{cycles}) containing $(n_b n_s)N_{cycles}$ subdivisions, plus or minus the desired *eḍuppu* offset of n_{shift} subdivisions.

Design 2: Cumulatively Progressive *Uttarārdha*s

[46] Our second design introduces cumulative progression into the *uttarārdha*. The *uttarārdha* is based on design U_2 (see Table 4), but incorporates cumulative growth over the three *kōrvai*s. Figure 13 and Table 6 show the structures of the new sub-compositions. The introductory pattern is simple and consists of three equal, repeating phrases.

[47] Following a similar approach as for Design 1, we get the constraint equation $\boldsymbol{9a_1 + 18x_1 + 12x_2 + 8x_3 = (n_b n_s)N_{cycles} + n_{shift}}$.

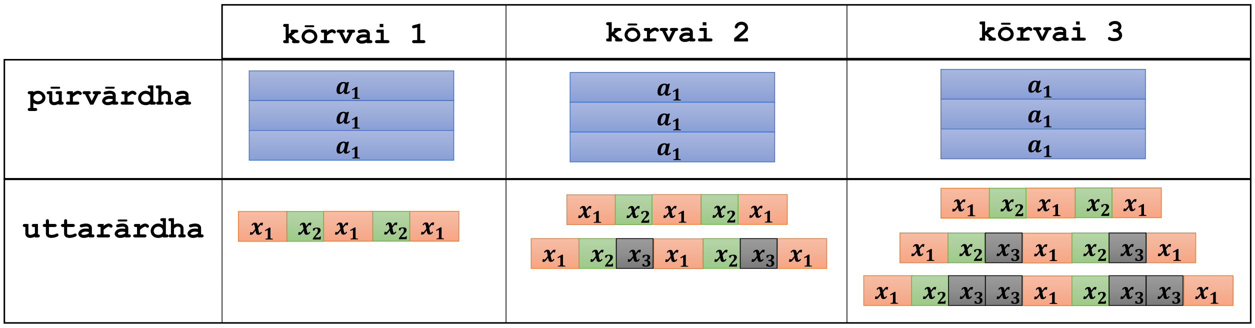


Figure 13: A geometric representation of the *pūrvārdha* and *uttarārdha* of the three *kōrvai*s of the second newly proposed rhythmic composition structure (Design 2).

Kōrvai #	*Pūrvārdha* pattern (in rhythmic subdivision counts)	*Uttarārdha* pattern (in rhythmic subdivision counts)
1	$3a_1$	$3x_1 + 2x_2$
2	$3a_1$	$\big(3x_1 + 2x_2, 3x_1 + 2(x_2 + x_3)\big)$
3	$3a_1$	$\big(3x_1 + 2x_2, 3x_1 + 2(x_2 + x_3), 3x_1 + 2(x_2 + 2x_3)\big)$
Total subdivisions	$9a_1$	$18x_1 + 12x_2 + 8x_3$

Table 6: Subdivision totals for the *pūrvārdha* (introductory part) and *uttarārdha* (concluding part) of the three *kōrvai*s forming the Design 2 rhythmic composition trio.

[48] The introduction of the progressively growing intermediary component x_3 into the *uttarārdha* contributes a total of $8x_3$ subdivisions across the *kōrvai* trio, which is not always a multiple of three and hence can potentially yield solutions to the challenges involving the $C_{3,4}$ rhythmic cycle with *eḍuppu* $n_{shift} = \pm 2$.

Concept Exemplification with 3-Beat Rhythmic Cycles

[49] In this subsection, we exemplify the algorithmic construction of *kōrvai*s based on Designs 1 and 2 above for the $C_{3,4}$ rhythmic cycle with $n_{shift} = \pm 2$ *eḍuppu*s. The general procedure involves setting up a linear multivariable equation where the independent variables are the *kōrvai* components and the dependent variable, for which we solve, is N_{cycles}, the number of complete rhythmic cycles consumed by the three *kōrvai*s when the n_{shift} offset is excluded. This equation is solved, and acceptable solutions are extracted algorithmically through a simple code written and executed using MATLAB. Carnatic vocal demonstrations of the examples that follow in Figures 14–17 are provided in the Appendix A2. These include sung performances (Video Examples 1, 2, 3, and 4) of the designs, showing how these *kōrvai*s might be used in non-percussive contexts, and annotated transcriptions in *sargam* and Western staff notation.

Exemplification of Design 1 for the $C_{3,4}$ Rhythmic Cycle with 2-Subdivision *Eḍuppu*s

[50] Applying Design 1 to *rūpaka tāḷa* ($C_{3,4}$) with an *eḍuppu* of $n_{shift} = \pm 2$, we get $\mathbf{18a_1 + 4a_2 + 9x_1 + 6x_2 = 12N_{cycles} \pm 2}$.

In the first case where $n_{shift} = +2$, the rhythmic composition will be two subdivisions longer than N_{cycles} rhythmic cycles, and in the case where $n_{shift} = -2$, the rhythmic composition will be two subdivisions shorter than the N_{cycles} rhythmic cycles.

The adopted algorithm is as follows:

Step 1: Prescribe desired values for the *uttarārdha* parameters x_1 and x_2.

Step 2: Prescribe a range of positive integer values for the *pūrvārdha* parameters a_1 and a_2.

Step 3: Solve for the unknown N_{cycles} and obtain the set of all solutions S for the prescribed range of the independent variables. We represent the i^{th} element of this set by $s_i = [a_1(i), a_2(i), x_1(i), x_2(i), N_{cycles}(i)]$, so that s_i is an array carrying the prescribed and solved parameters for the *kōrvai.*

Step 4: Obtain the subset of solutions S_{int} such that N_{cycles} is a positive integer. Specifically, $S_{int} = \left\{ s_i | \; s_i \in S \text{ and } N_{cycles}(i) = \text{floor}\left(N_{cycles}(i)\right)\right\}$, where $\text{floor}(x)$ is the greatest integer that is less than or equal to x.

Step 5: Compose the *kōrvai* with structure determined by a_1, a_2, x_1, and x_2 from a chosen S_{int} solution.

[51] For illustrative purposes, we set $x_1 = 18,\; x_2 = 1$ as our *uttarārdha* values in Step 1.

[52] We set the range of a_1 and a_2, our *pūrvārdha* values, as $R_{a_1 a_2} = \{a_1, a_2 \in \boldsymbol{Z}: 1 \leq a_1 \leq 8, 1 \leq a_2 \leq 5\}$, where $\boldsymbol{Z}$ is the set of integers. The values of a_1 and a_2 that lead to an integer solution to N_{cycles} are tabulated in Table 7 for both cases, $n_{shift} = \pm 2$.

[53] The shaded rows in Table 7 are the solutions that we will demonstrate via musical examples. Specifically, sample rhythmic compositions corresponding to these solutions for *eḍuppu*s $n_{shift} = 2$ and $n_{shift} = -2$ are notated below in Figure 14 and Figure 15, respectively (again, see Appendix A2 for vocal recordings and annotated transcriptions). Especially note the role of the light blue boxes, which represent the novel a_2 phrases in the *pūrvārdha*.

Solution #	Case 1: $\boldsymbol{n_{shift}} = \boldsymbol{+2}$			Case 2: $\boldsymbol{n_{shift}} = \boldsymbol{-2}$		
	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{N_{cycles}}$	$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{N_{cycles}}$
1	1	2	16	1	1	16
2	1	5	17	1	4	17
3	3	2	19	3	1	19
4	3	5	20	3	4	20
5	5	2	22	5	1	22
6	5	5	23	5	4	23
7	7	2	25	7	1	25
8	7	5	26	7	4	26

Table 7: Acceptable solutions to *kōrvai*s based on Design 1 for $C_{3,4}$ rhythmic cycles, with $n_{shift} = \pm 2$. The values of x_1 and x_2 are prescribed as 18 and 1 for demonstrative purposes, and are not shown.

Cycle#	Beat 1				Beat 2				Beat 3			
1	tha	.	dhim	.	.	tha	.	dhim	.	.	tha	.
2	dhim	.	.	tha	dhim	.	gi	na	thom	tha	dhim	.
3	gi	na	thom	tha	dhim	.	gi	na	thom	.	tha	dhim
4	.	gi	na	thom	tha	dhim	.	gi	na	thom	tha	dhim
5	.	gi	na	thom	.	tha	dhim	.	gi	na	thom	tha
6	dhim	.	gi	na	thom	tha	dhim	.	gi	na	thom	tha
7	.	dhim	.	.	tha	.	dhim	.	.	tha	.	dhim
8	.	.	tha	ka	tha	.	dhim	.	.	tha	.	dhim
9	.	.	tha	.	dhim	.	.	tha	dhim	.	gi	na
10	thom	tha	dhim	.	gi	na	thom	tha	dhim	.	gi	na
11	thom	.	tha	dhim	.	gi	na	thom	tha	dhim	.	gi
12	na	thom	tha	dhim	.	gi	na	thom	.	tha	dhim	.
13	gi	na	thom	tha	dhim	.	gi	na	thom	tha	dhim	.
14	gi	na	thom	tha	.	dhim	.	.	tha	.	dhim	.
15	.	tha	.	dhim	.	.	tha	ka	tha	.	dhim	.
16	.	tha	.	dhim	.	.	tha	.	dhim	.	.	tha
17	ka	dhi	na	tha	.	dhim	.	.	tha	.	dhim	.
18	.	tha	.	dhim	.	.	tha	dhim	.	gi	na	thom
19	tha	dhim	.	gi	na	thom	tha	dhim	.	gi	na	thom
20	.	tha	dhim	.	gi	na	thom	tha	dhim	.	gi	na
21	thom	tha	dhim	.	gi	na	thom	.	tha	dhim	.	gi
22	na	thom	tha	dhim	.	gi	na	thom	tha	dhim	.	gi
23	na	thom										

Figure 14: *Kōrvai* trios based on Design 1 for the $C_{3,4}$ rhythmic cycle (*rūpaka tāḷa*) with an *anāgata eḍuppu*, $n_{shift} = +2$. The composition starts on the first subdivision of the first beat and ends two subdivisions into the first beat. Dark blue color indicates the introductory pattern measuring $3a_1$ subdivisions. Light blue indicates the intervals a_2 and $2a_2$ introduced in the *pūrvārdha* of the second and the third *kōrvai*s, respectively, as per Design 1. The yellow and red cells indicate the x_1 and x_2 portions of the *uttarārdha*, respectively. The composition finishes in the 23rd rhythmic cycle (after completing 22 complete cycles).

Cycle#	Beat 1				Beat 2				Beat 3			
1	tha	.	dhim	.	.	tha	.	dhim	.	.	tha	.
2	dhim	.	.	tha	dhim	.	gi	na	thom	tha	dhim	.
3	gi	na	thom	tha	dhim	.	gi	na	thom	.	tha	dhim
4	.	gi	na	thom	tha	dhim	.	gi	na	thom	tha	dhim
5	.	gi	na	thom	.	tha	dhim	.	gi	na	thom	tha
6	dhim	.	gi	na	thom	tha	dhim	.	gi	na	thom	tha
7	.	dhim	.	.	tha	.	dhim	.	.	tha	.	dhim
8	.	.	tha	ka	dhi	na	tha	.	dhim	.	.	tha
9	.	dhim	.	.	tha	.	dhim	.	.	tha	dhim	.
10	gi	na	thom	tha	dhim	.	gi	na	thom	tha	dhim	.
11	gi	na	thom	.	tha	dhim	.	gi	na	thom	tha	dhim
12	.	gi	na	thom	tha	dhim	.	gi	na	thom	.	tha
13	dhim	.	gi	na	thom	tha	dhim	.	gi	na	thom	tha
14	dhimi	.	gi	na	thom	tha	.	dhim	.	.	tha	.
15	dhim	.	.	tha	.	dhim	.	.	tha	ka	dhi	na
16	tha	.	dhim	.	.	tha	.	dhim	.	.	tha	.
17	dhim	.	.	tha	tha	ki	ta	tha	ka	dhi	na	tha
18	.	dhim	.	.	tha	.	dhim	.	.	tha	.	dhim
19	.	.	tha	dhim	.	gi	na	thom	tha	dhim	.	gi
20	na	thom	tha	dhim	.	gi	na	thom	.	tha	dhim	.
21	gi	na	thom	tha	dhim	.	gi	na	thom	tha	dhim	.
22	gi	na	thom	.	tha	dhim	.	gi	na	thom	tha	dhim
23	.	gi	na	thom	tha	dhim	.	gi	na	thom		

Figure 15: *Kōrvai* trios based on Design 1 for the $C_{3,4}$ rhythmic cycle (*rūpaka tāḷa*) with an *atīta eḍuppu*, $n_{shift} = -2$. The composition starts on the first subdivision of the cycle and ends two subdivisions before the end of the rhythmic cycle. Dark blue indicates the introductory pattern measuring $3a_1$ subdivisions. Light blue indicates the intervals a_2 and $2a_2$ in the *pūrvārdha* of the second and the third *kōrvai*s, respectively, as per Design 1. The yellow and red cells indicate the x_1 and x_2 portions of the *uttarārdha*, respectively. The composition finishes in the 23rd rhythmic cycle (after completing 23 complete cycles minus two subdivisions).

Exemplification of Design 2 for the $C_{3,4}$ Rhythmic Cycle with 2-Subdivision *Eḍuppus*

[54] For the rhythmic cycle $C_{3,4}$ and $n_{shift} = \pm 2$, we have the constraint equation $\mathbf{9a_1 + 18x_1 + 12x_2 + 8x_3 = 12N_{cycles} \pm 2}$.

The algorithm here works as follows:

Step 1: Choose desired values for a_1, x_2, and x_3. Recall that a_1 is the length of the basic phrase defining the *pūrvārdha*, while x_2 and x_3 are the intermediary phrase lengths in the *uttarārdha*.

Step 2: Prescribe a range of positive integer values for the initial phrase, x_1, of the *uttarārdha*.

Step 3: Solve for the unknown N_{cycles} and obtain the set of all solutions S for the prescribed range of the independent variable x_1. We will represent the i^{th} element of this set as $s_i = [a_1(i), x_1(i), x_2(i), x_3(i), N_{cycles}(i)]$, so that s_i is an array containing the prescribed and solved parameters of the *kōrvai.*

Step 4: Obtain the subset of solutions S_{int} for which N_{cycles} is a positive integer. Specifically, $S_{int} = \left\{ s_i |\, s_i \in S \text{ and } N_{cycles}(i) = \text{floor}\left(N_{cycles}(i)\right)\right\}$, where $\text{floor}(x)$ is the greatest integer that is less than or equal to x.

Step 5: Compose the *kōrvai* with structure determined by a_1, x_1, x_2, and x_3 from a chosen S_{int} solution.

For our illustration, we choose $a_1 = 8$, $x_2 = 0$, and $x_3 = 1$ for $n_{shift} = +2$, and $x_3 = 2$ for $n_{shift} = -2$. We obtain the acceptable solutions given in Table 8 for x_1 in the range $R_{x_1} = \{x_1 \in \boldsymbol{Z}: 5 \leq x_1 \leq 9\}$.

Figures 16 and 17 then adapt the highlighted solutions from Table 8 to form sample rhythmic compositions (see Appendix A2 for recordings and transcriptions).

Solution #	Case 1: $n_{shift} = 2$		Case 2: $n_{shift} = -2$	
	x_1	N_{cycles}	x_1	N_{cycles}
1	5	14	5	15
2	7	17	7	18
3	9	20	9	21

Table 8: Acceptable solutions to *kōrvai*s based on Design 2 for $C_{3,4}$ rhythmic cycles, with $n_{shift} = \pm 2$. The values of a_1, x_2, and x_3 are prescribed as $a_1 = 8$, $x_2 = 0$, and $x_3 = 1$ (for $n_{shift} = +2$) and $x_3 = 2$ (for $n_{shift} = -2$) for demonstrative purposes, and are not shown in the table.

Cycle#	Beat 1				Beat 2				Beat 3			
1	tha	ka	dhi	na	.	dhim	.	.	tha	ka	dhi	na
2	.	dhim	.	.	tha	ka	dhi	na	.	dhim	.	.
3	tha	dhim	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi
4	gi	na	thom	tha	ka	dhi	na	.	dhim	.	.	tha
5	ka	dhi	na	.	dhim	.	.	tha	ka	dhi	na	.
6	dhim	.	.	tha	dhi	gi	na	thom	tha	dhi	gi	na
7	thom	tha	dhi	gi	na	thom	tha	dhi	gi	na	thom	.
8	tha	dhi	gi	na	thom	.	tha	dhi	gi	na	thom	tha
9	ka	dhi	na	.	dhim	.	.	tha	ka	dhi	na	.
10	dhim	.	.	tha	ka	dhi	na	.	dhim	.	.	tha
11	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi	gi
12	na	thom	tha	dhi	gi	na	thom	.	tha	dhi	gi	na
13	thom	.	tha	dhi	gi	na	thom	tha	dhi	gi	na	thom
14	.	.	tha	dhi	gi	na	thom	.	.	tha	dhi	gi
15	na	thom										

Figure 16: *Kōrvai* trios based on Design 2 for the $C_{3,4}$ rhythmic cycle (*rūpaka tāḷa*) with an *anāgata eḍuppu*, $n_{shift} = +2$. The composition starts on the first subdivision of the cycle and ends two subdivisions into the first beat of the cycle. Blue indicates the *pūrvārdha,* yellow indicates the $(3x_1 + 2x_2)$-subdivision building block of the *uttarārdha,* and red cells indicate the x_3 component of the *uttarārdha*, respectively. The composition finishes in the fifteenth rhythmic cycle (after completing fourteen complete cycles).

Cycle#	Beat 1				Beat 2				Beat 3			
1	tha	ka	dhi	na	.	dhim	.	.	tha	ka	dhi	na
2	.	dhim	.	.	tha	ka	dhi	na	.	dhim	.	.
3	tha	dhim	gi	na	thom	tha	dhi	gi	na	thom	tha	dhi
4	gi	na	thom	tha	ka	dhim	na	.	dhim	.	.	tha
5	ka	dhi	na	.	dhim	.	.	tha	ka	dhi	na	.
6	dhim	.	.	tha	dhi	gi	na	thom	tha	dhi	gi	na
7	thom	tha	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha
8	.	tha	dhi	gi	na	thom	tha	.	tha	dhi	gi	na
9	thom	tha	ka	dhi	na	.	dhim	.	.	tha	ka	dhi
10	na	.	dhim	.	.	tha	ka	dhi	na	.	dhim	.
11	.	tha	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha
12	dhi	gi	na	thom	tha	dhi	gi	na	thom	tha	.	tha
13	dhi	gi	na	thom	tha	.	tha	dhi	gi	na	thom	tha
14	dhi	gi	na	thom	tha	.	dhi	.	tha	dhi	gi	na
15	thom	tha	.	dhi	.	tha	dhi	gi	na	thom		

Figure 17: *Kōrvai* trios based on Design 2 for the $C_{3,4}$ rhythmic cycle (*rūpaka tāḷa*) with an *atīta eḍuppu*, $n_{shift} = -2$. Note that the composition starts on the first subdivision of the cycle and ends two subdivisions before the end of the cycle, as required. Blue indicates the introductory pattern. The yellow cells indicate the $(3x_1 + 2x_2)$-subdivision building block of the *uttarārdha* and red cells indicate the x_3 components introduced in the *uttarārdha*, respectively. The composition finishes in the fifteenth rhythmic cycle (after completing fifteen complete cycles minus two subdivisions corresponding to the *eḍuppu*).

Generalizing the Approach

[55] A generalized approach to the above designs would involve a combination of artistry and algorithmic thinking, as graphically depicted in Figure 18. The process steps highlighted in blue can be implemented through software.

Compositional Applications Beyond Carnatic Music

Introduction and Motivations

[56] The demonstrations in the previous section have exclusively considered our new rhythmic designs in the context of Carnatic music. Indeed, within Carnatic music, these novel theories offer innumerable new possibilities for the solo vocalist or instrumentalist. An interesting question, though, is whether these theories about *kōrvai* structure could have applications beyond South Indian classical music.

[57] A primary motivation for the latter question is that Carnatic techniques have already been applied to jazz and funk improvisation (Krishnamurthy 2015), jazz-based art music (Young 2015; Okazaki 2006; Takeno 2017), Indian hip hop (Akundi 2021), and other examples of musical fusion. Additionally, numerous musicians and composers have explored how Indian music could interact with Western classical music. Examples include Olivier Messiaen's adaptation of Śārṅgadeva's (1175-1247) *deśitāḷa*s to rhythms in his contemporary compositions

(Šimundža 1987), and Philip Glass's Indian-inspired works such as the opera *Satyagraha* and the oratorio *The Passion of Ramakrishna*, influenced by a formative period when Glass worked as sitarist Ravi Shankar's assistant (Glass 2015). A more theoretically based approach to Indian/Western classical fusion comes from Chitravina Ravikiran and Robert Morris, whose concept of "melharmony" allows Carnatic *rāga*s to be harmonized using notes of the *rāga*, in a manner that respects the inherent properties of individual *rāga*s (Morris and Ravikiran 2006; Ravikiran 2014).

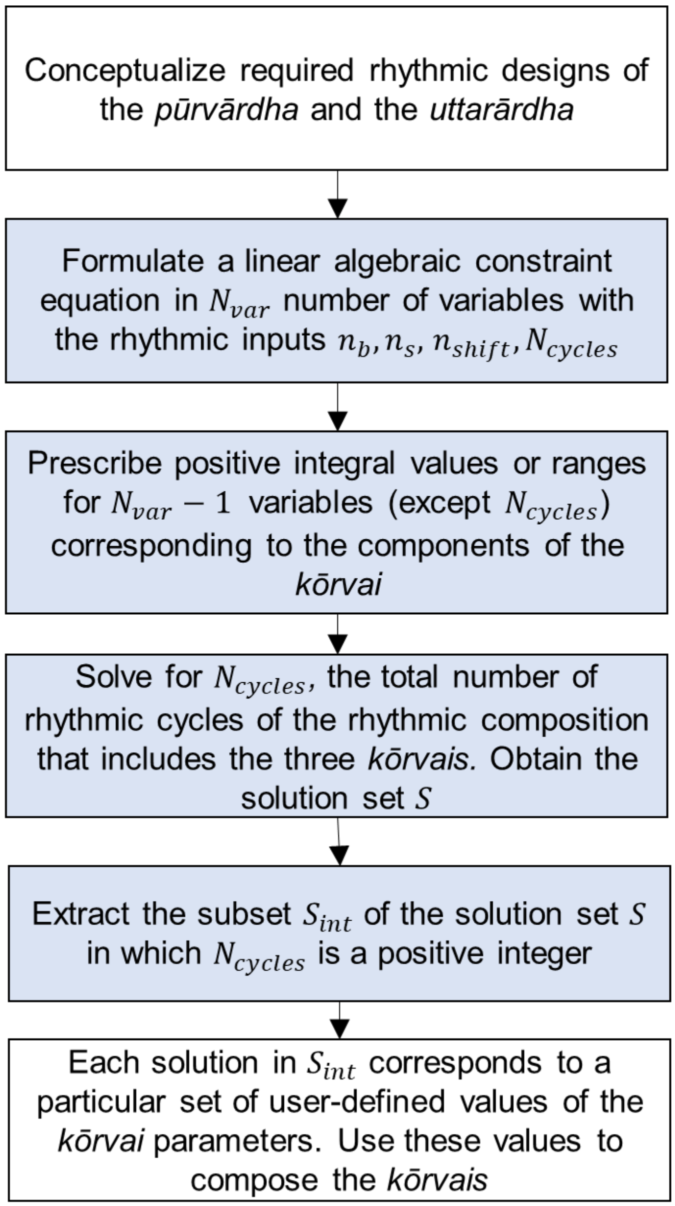


Figure 18: A generic process flow for algorithmic *kōrvai* composition in any *tāḷa* (C_{n_b,n_s} rhythmic cyle) for a required *eḍuppu* given by n_{shift}.

[58] These examples strongly suggest that the current research on *kōrvai* structures could have applications to other musical styles. Indeed, since the *kōrvai* is a rhythmic/metric structure, it could be used to ground a wide variety of melodic, harmonic, and timbral combinations and uses. An apparent challenge, though, is that our discussion thus far has depended on assumptions about musical structure specific to Carnatic music-making, as demonstrated by the *kōrvai* models and constraints from paragraphs [26]–[29] above. How could the current research be applicable in musical situations in which these constraints are no longer assumed, such as contemporary Western art music where compositional aesthetics can vary widely? The next subsection considers this question in detail.

Compositional Philosophy

[59] Applying this study's new *kōrvai* theories to other musical styles requires careful consideration of compositional philosophy. Namely, the *kōrvai* models and constraints described previously, such as the principle of arithmetic rhythmic progression between sub-compositions and the requirement that a *kōrvai* trio end exactly where a referential melody begins, are fundamental to Carnatic rhythm but absent from many other musical traditions. As such, if the *kōrvai* constraints are to apply in these new contexts, they must be externally imposed. This imposition of constraints would be for artistic purposes rather than traditional precedent.

[60] A non-Carnatic composer or performer may wonder why we might impose such constraints on a composition or improvisation. It is important to acknowledge, though, that composition and improvisation already bear implicit constraints, even in seemingly open-ended Western styles. Jazz, for instance, has an air of freedom, but improvising soloists are constrained by the chord changes given in the lead sheet and rules about what melodic notes "agree" with a given chord. Atonal art music from the first half of the twentieth century is not obligated to follow the harmonic rules of earlier eras, but conventional instrumental timbres and equally-tempered tuning are often assumed.[18]

[61] Moreover, in the early twentieth century, atonal composers began turning to serial techniques, using mathematical principles to structure melody, harmony, and contrapuntal lines. These techniques allowed composers to organize their new musical sounds using externally imposed structures, thus illustrating the powerful role that imposed constraints might play in certain Western compositional or improvisational contexts.[19] By extension, the constraints inherent in Carnatic music could be intentionally and strategically applied to composition in other musical realms, even if the non-rhythmic parameters such as melody, harmony, and timbre vary substantially from Indian classical norms.

[62] This said, composers and improvisers interested in applying Carnatic principles to non-Carnatic music should avoid doing so haphazardly. The authors of the current study strongly recommend careful study of traditional Indian classical music in any such endeavor, which would ideally involve not just reading about Carnatic music theory, but listening to real musical

performances and having conversations with practitioners of Carnatic music. In this way, a composer can explore new compositional territory while respecting the deep musical traditions undergirding these compositional constraints.

Applying the Theories in Non-Carnatic Settings

General Strategies

[63] There are many ways one might adapt Carnatic principles to other musical idioms. The goals of the forthcoming demonstrations, however, are specific and limited: we aim to apply the four constraints from paragraph [27], and the new *kōrvai* structures of paragraphs [39]–[55], to composing Western-classical-style music. Numerous other adaptations of Carnatic principles are possible, but are beyond the scope of this limited study.

[64] To incorporate our novel *kōrvai* structures into a Western classical composition, the first necessary component is a distinct musical theme that can serve as a point of departure. In Carnatic music, this would be a referential musical line to which a composition consistently returns. As discussed previously, the point in the *tāḷa* where this referential line begins defines the *eḍuppu*, which serves as a "landing point" for a *kōrvai* trio—i.e., the last *kōrvai* must end exactly where the referential phrase begins.

[65] The calculations in paragraphs [49]–[55] investigated *eḍuppu*s that are located some nonzero distance from the start of a *tāḷa* cycle. As such, in the compositional demonstrations to follow, we opt to begin our main themes on rhythmic locations other than the downbeat. The starting rhythmic location should always be chosen carefully and intentionally, as even a slight change in this location can have a drastic effect on the possible *kōrvai* structures.

[66] Once the main theme has been composed, a composer can consider the broad scope of the composition, including the overall length, number of sections, and so forth. Within this compositional plan, the main theme, after being stated early in the composition, should ideally return at least once. Then, any time this theme returns, the composer can precede it with a *kōrvai*, thereby generating rhythmic tension and interest leading to the theme's entrance. For instance, in a jazz-influenced composition, the *kōrvai* could lead from an instrumental solo to the return of the head. In a movement in classical sonata form, the *kōrvai* might build intensity at the end of the development section and carry the music smoothly into the recapitulation. In a solo percussion piece, the *kōrvai* might lead to the return of an opening groove.

[67] Regardless of the specific musical style or form, the rhythmic structure of the *kōrvai* can be determined using the method given in Figure 18. Namely, user-defined values can dictate the length of each phrase/section of the *kōrvai*, and once these are set, composition can begin. A new question then emerges: how closely should the compositional use of the *kōrvai* align with Carnatic norms?

[68] In particular, we might envision a spectrum from close adherence to Carnatic principles to much looser adherence. At the first end of the spectrum could be melodically driven music that incorporates traditional Carnatic rhythmic patterns and approximates melodic motions within a specific Carnatic *rāga*.[20] At the other end, we might have a composition that implements *kōrvai* structures to depart from an established meter, but does not attempt to incorporate accepted Carnatic rhythmic patterns, melodic phrases, or *rāga*s. Between these two extremes lie various shadings. For instance, a *kōrvai* might incorporate typical Carnatic rhythmic patterns but use non-Carnatic scales or modes instead of *rāga*s. A *kōrvai* trio could even start at one end of the spectrum and move to the other as the music progresses—for instance, by starting fairly traditionally, and deviating from Carnatic expectations as the return of the main theme approaches.

[69] A composer could also adapt the *kōrvai* structure to precede a musical theme that is *not* the main theme. The composer would need to recognize, though, that this would be a more significant departure from Carnatic tradition. Again, ethical considerations are warranted here, and composers should be careful, when departing from Carnatic norms, that they do not do so thoughtlessly. The better understanding one has of Carnatic music, the more effectively and respectfully they can incorporate *kōrvai* structures into other musical contexts.

Examples

[70] To illustrate the above principles, we provide two musical examples illustrating how our novel *kōrvai* structures might be used in non-Carnatic settings. These obviously only provide a preliminary taste of how these *kōrvai*s could be applied to other musical styles, but they are valuable in suggesting possible routes a composer might follow.

[71] In the previous subsection, we argued that applications of Carnatic techniques to non-Carnatic music conceptually occur on a continuum: at one extreme is music that departs very little from South Indian classical traditions, while the other end contains music that only loosely draws on Carnatic norms. Figure 19 (and Audio Examples 1 and 2) present two excerpts from a hypothetical solo oboe composition that stays, in certain respects, fairly close to Carnatic expectations.

[72] This would-be composition is based on the Western musical theme *La Folia*, shown in Figure 19a. After stating the main theme, the composer could proceed in numerous directions, such as a set of variations on *La Folia* or a series of contrasting sections.

[73] In either case, suppose the composer eventually pulls the music into a state of increased virtuosity and musical tension, in preparation for a triumphant return of the original *La Folia* theme. As shown in Figure 19b, such a section could easily house a *kōrvai* structure. Because the main theme starts with an eighth-note pickup, or two sixteenth-note subdivisions, we incorporate a *kōrvai* design where $n_{shift} = -2$.

Figure 19: Hypothetical oboe composition: opening theme and *kōrvai* implementation.

[74] This *kōrvai* trio stays close to Carnatic expectations, at least from a rhythmic standpoint, as the rhythms are based on traditional *solkaṭṭu* patterns. In particular, these rhythms are taken directly from Design 2, $n_{shift} = -2$ (Figure 17). Melodically, however, the excerpt is not based on an Indian classical *rāga*, but the Western A minor scale.[21] Furthermore, the melody suggests Western harmonic expectations due to the arpeggiated chords and sequential melodic figures.

Nevertheless, the *kōrvai* concludes in a quasi-Carnatic manner, ending precisely where the *La Folia* theme returns and marking the conclusion of a difficult, highly technical section of the piece.

[75] Figures 20–21 (and Audio Examples 3 and 4) present a different approach to adapting our new *kōrvai* structures to non-Carnatic music. Instead of a melodic composition for a solo wind instrument, this example consists of a harmonically driven piece for piano solo.

[76] The main theme of the piece, shown in Figure 20a, starts two eighth notes before the downbeat. Thus, we again have an $n_{shift} = -2$ scenario, albeit in a different notated meter. Unlike the oboe solo, this hypothetical composition has multiple simultaneous musical lines, creating a sound world of contemporary harmony and counterpoint. Already, the primacy of harmony and changing chords, rather than melody, makes this piece decided less Carnatic in character than the oboe piece. Additionally, rather than being based on a single scale and pitch center, this piece implements two different Western modes, each with a different tonic note (namely, D Dorian and E-flat Lydian).

[77] Pushing the composition even further from Carnatic norms is the *kōrvai* trio shown in Figures 20b–21. While all main sections of the *kōrvai* are still present, the *pūrvārdha* rhythms from Figure 17 have been placed in the left hand of the piano. As such, rather than being soloistically foregrounded, these rhythms are initially pushed to the background. The right hand, meanwhile, plays a series of long chords over the *pūrvārdha* rhythms.

[78] In the *uttarārdha*, the musical dynamic changes. Neither hand attempts to incorporate rhythms from Figure 17; instead, we have new rhythmic phrases spanning the desired number of subdivisions. Moreover, the melodic gestures in the right hand incorporate rhythmic values that are faster than the *kōrvai*'s minimal rhythmic value (here represented by the eighth note), thereby subverting constraint 4 from paragraph [27]. Finally, and more holistically, the scalar and chordal techniques used here are idiomatic to the piano, giving the piece a sound distinct from that of classical Carnatic instrumental playing.

[79] In sum, this piano composition incorporates the *kōrvai* structure much more freely than the oboe piece, and is therefore further removed from Carnatic tradition. In both hypothetical compositions, though, the spirit of the *kōrvai* as forming a rhythmically intricate ending to a section and leading back to a recognizable melody is retained. Moreover, imposing the new *kōrvai*s as a musical constraint in these decidedly non-Carnatic pieces has paved the way to novel compositional possibilities. When used in such contexts, these *kōrvai* structures create an idiosyncratic fusion of Indian and Western classical aesthetics and thus represent a unique form of intercultural dialogue.

Figure 20: Hypothetical piano composition: opening theme and start of *kōrvai*.

Figure 21: End of *kōrvai* leading to *eḍuppu* in hypothetical piano piece.

Summary and Outlook

[80] We have demonstrated a key step toward algorithmic rhythmic composition in the context of Carnatic music through a systematic approach involving the following: (a) clarifying, and mathematically modeling, essential rhythmic constructs and concepts and their place in the scheme of a Carnatic music concert; (b) highlighting the limitations of existing traditional approaches to rhythmic composition in Carnatic music; (c) conceptualizing and algorithmically determining new rhythmic designs to address the limitations of traditional rhythmic constructs; and (d) musically demonstrating the proposed designs in the context of Carnatic percussive/vocal music and hypothetical non-Carnatic compositions. We believe that this generative approach to *kōrvai* structure and substructure, and the approach's potential extensions, can lead to significant strides in rhythmic compositional complexity in Carnatic music and beyond.

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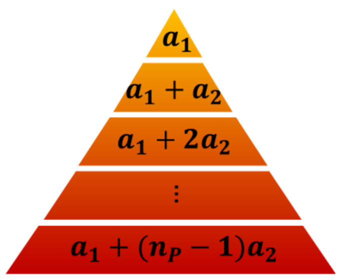
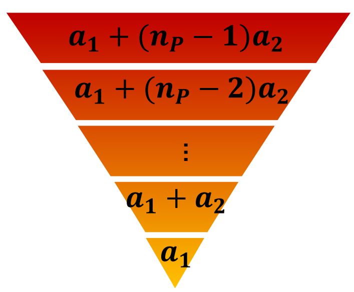
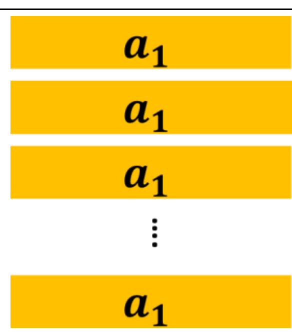
Appendix A1: Detailed Mathematical Descriptions of Various Traditional *Pūrvārdha* and *Uttarārdha* Structures

Tables A1 and A2 show detailed mathematical representations of the traditional *pūrvārdha* and *uttarārdha* structures discussed in Tables 3 and 4. These formulations illustrate the number of subdivisions in each constituent rhythmic phrase and in each overall *pūrvārdha* and *uttarārdha* structure.

Design #	Illustration	Total subdivisions
P_1		$n_P a_1 + \frac{n_P(n_P-1)}{2} a_2$, where the main underlying melodic/rhythmic ideas span a_1 and a_2 subdivisions, respectively, and n_p is the number of lines in the cumulative pattern.
P_2		$n_P a_1 + \frac{n_P(n_P-1)}{2} a_2$, where a_1, a_2, and n_p have the same meanings as above.
P_3		$n_p a_1$, where n_p is the number of times the pattern with a_1 subdivisions is repeated

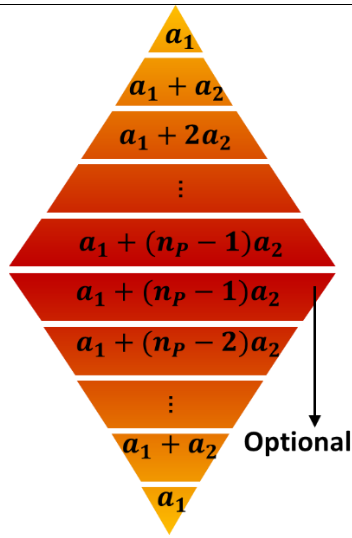
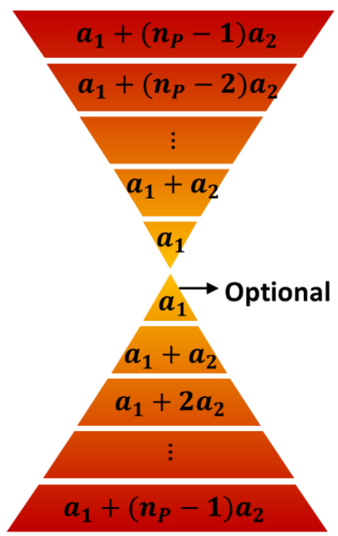
<table>
<tr>
<td>P_4</td>
<td>
</td>
<td>$2\left(n_P a_1 + \frac{n_P(n_P-1)}{2} a_2\right) - c(a_1 + (n_P - 1)a_2)$,
where $c = 1$ if the n_P^{th} line repeats at the start of the shrinking subsequence and is 0 if it is omitted.</td>
</tr>
<tr>
<td>P_5</td>
<td>
</td>
<td>$2\left(n_P a_1 + \frac{n_P(n_P-1)}{2} a_2\right) - c a_1$,
where $c = 1$ if the n_P^{th} line repeats at the start of the shrinking subsequence and is 0 if it is omitted.</td>
</tr>
</table>

Table A1. Detailed mathematical representations of traditional *pūrvārdha* structures.

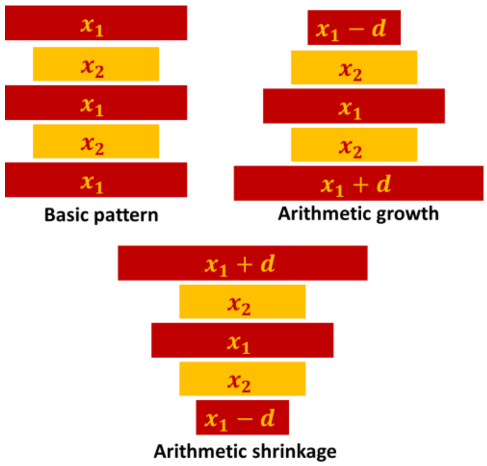
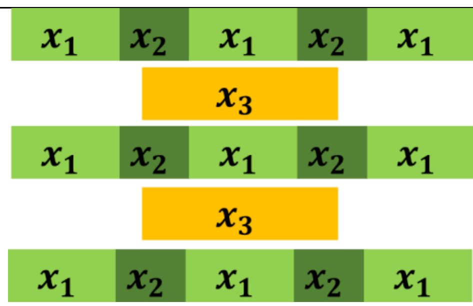
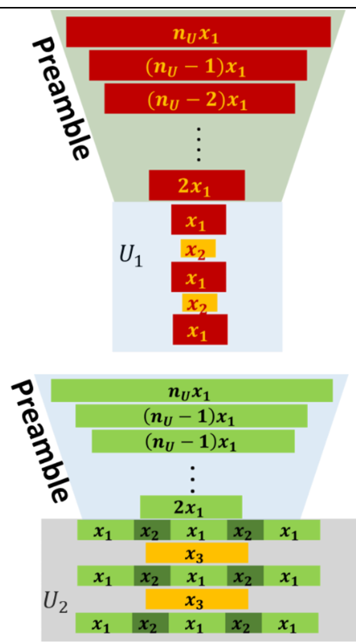
Design #	**Illustration**	**Total subdivisions**
U_1		$3x_1 + 2x_2$
U_2		$3(3x_1 + 2x_2) + 2x_3$
U_3		**Case 1**: When the final pattern is $U_1: \frac{n_U(n_U-1)}{2}x_1 + 3x_1 + 2x_2$ **Case 2**: When the final pattern is $U_2: \frac{n_U(n_U-1)}{2}x_1 + 3(3x_1 + 2x_2) + 2x_3$

Table A2. Detailed mathematical representations of traditional *uttarārdha* structures.

Appendix A2: Vocal *Kōrvai* Transcriptions (See Video Examples 1, 2, 3, 4)

Design 1, n_{shift} = -2 (Video Example 1, 1:44-2:41)
Based on "Hiraṇmayīm Lakṣmīm" (Muttusvāmi Dīkṣitar)
Tāḷa: Rūpaka; Rāga: Lalitha
Reference Phrase: sangīta vādya vinōdinīm (ćaranam)

***Note:** C on the staff has arbitrarily been chosen to represent "sa." Bold swaras mark the start of a new melodic/ rhythmic subpattern.

Design 2, n_{shift} = -2 (Video Example 2, 1:08-1:45)
Based on "Hiraṇmayīm Lakṣmīm" (Muttusvāmi Dīkṣitar)
Tāḷa: Rūpaka; Rāga: Lalithā
Reference Phrase: sangīta vādya vinōdinīm (ćaranam)

Design 1, n_{shift} = +2 (Video Example 3, 1:46-2:44)
Based on "Rāju veḍale jūtāmu rāre" (Tyāgarāja)
Tāḷa: Rūpaka; Rāga: Tōḍi
Reference Phrase: kāvērī tīramunanu (ćaranam)

Design 2, n_{shift} = +2 (Video Example 4, 1:02-1:39)
Based on "Rāju veḍale jūtāmu rāre" (Tyāgarāja)
Tāḷa: Rūpaka; Rāga: Tōḍi
Reference Phrase: kāvērī tīramunanu (ćaranam)

NOTES

1. Most commonly used *tāḷa*s, such as the *suḷādi sapta tāḷa*s and the *cāpu tāḷa*s, have subdivisions of equal duration. However, there are customized "composite" *tāḷa*s involving multiple *naḍai*s (a concept discussed in paragraphs [16]–[22]) where subdivisions within different beats of a single *āvartana* are of different durations. Such *tāla*s are not within the purview of our current investigation.
2. For a given musical composition, t_0 is an artistic choice that depends on the mood of a concert, capabilities of the artist, and the meaning and emotional context of the composition.
3. There are conflicting views on the nature of the *cāpu tāḷa*s. For instance, while some artists define *miṣra cāpu tāḷa* as having seven equal beats, others argue for one long beat (three subdivisions) followed by two shorter beats (two subdivisions). In a conscious departure from Lerdahl and Jackendoff 1983, the authors take the latter view that *cāpu tāḷa* beats may vary in length; regarding non-isochronous beats, see Polak, London, and Jacoby 2016. More generally, throughout this article, we explicitly assume a culturally relativist, non-universalist perspective in which concepts such as "beat" and "meter" may have definitions, implications, and connotations that vary between musical cultures. Similarly, we acknowledge that ways of listening to, conceiving, and experiencing music may be culture-specific.
4. Changing the speed in Carnatic music is roughly analogous to augmentation or diminution in Western music. For instance, if the phrase "Sa---Ri---Ga---Ri---" lasts four beats in first speed, it will last two beats in second speed ("Sa-Ri-Ga-Ri-") and one beat in third speed ("SaRiGaRi"). See Nelson (2008, 16).
5. While many written descriptions of *caturaṣra naḍai* (especially in Western scholarship) describe it as having four subdivisions per beat (based on the Sanskrit word *caturaṣra*'s associations with the number four), the lead author's experience as a student and performer of Carnatic music suggests that this point is debatable. In particular, Carnatic vocalists and percussionists disagree about whether the primary subdivision of *caturaṣra naḍai* should be one or four. The authors' perspective, simply stated, is that if the total number of subdivisions per beat is divisible by two, then it is not the most fundamental speed (first speed). An exception to this is the *sampūrṇa khanḍa naḍai*, where the first speed implies ten subdivisions per beat. While this *naḍai* can be mathematically treated as *khanḍa naḍai* in the second speed, the authors prefer not to do so in the current study. In all other cases, the first speed corresponds to an odd integer that fundamentally represents the *naḍai*.
6. There is little clear documentation in the research literature for why *kōrvai*s are traditionally played in threes. From a practical standpoint, playing *kōrvai*s in threes gives co-percussionists about to play in the *tani āvartanam*, or the main artist preparing for the reference verse after the *tani āvartanam*, ample time to prepare for subsequent action. A possible technical rationale is that three is the smallest number of mathematical elements required to know what type of mathematical progression is occurring. For example, if the first two elements of a series are one and two, we do not know whether the progression is arithmetic or geometric, as the third element could be three or four, respectively.
7. Note that the constraints that follow distill commonly understood principles in Carnatic music. These principles have primarily been passed down orally from teacher to student, meaning there is limited written documentation for them. The lead author presents these principles based on extensive experience as a Carnatic musician and continuing studies with a senior percussionist.
8. A multi-*naḍai* paradigm is an open possibility that will be considered in later work.
9. Simply stated, our constraints 1 and 2 are common to all *kōrvai*s, while constraints 3 and 4 may be transgressed in certain contexts.
10. While Tables 3 and 4 describe essential Carnatic rhythmic structures, their musical implementation is not complete without optimal melodic phrasing or drum beat patterns that clearly demarcate rhythmic boundaries. In other words, pitch and timbre are imperative to conveying the underlying rhythmic ideas.
11. Specifically, if $n_b n_s$ is a multiple of three, then $K(n_b n_s)$ is also a multiple of three. Therefore, if n_{shift} is not a multiple of three, then $N_{total} = K(n_b n_s) + n_{shift}$ cannot be a multiple of three.
12. In contrast, consider a scenario where the total number of subdivisions per cycle in a *tāḷa* is *not* a multiple of three. An example is *ādi tāḷa* in the *viḷambha kāla* ($C_{16,4}$), which comprises sixteen beats, or sixty-four subdivisions, in the *caturaṣra naḍai* in third speed (*madhyama kāla*). Suppose one wants to use conventional structures to compose a *kōrvai* trio, and $n_{shift} = +4$—a non-multiple of three, and a reasonably frequent occurrence in concerts. In this case, a trio where each *kōrvai* is forty-four subdivisions long easily satisfies all constraints, as the total trio length is $44 \times 3 = 132$, which is equal to two complete *tāḷa* cycles plus four subdivisions. Thus, if we start on the first subdivision of a cycle, we will end precisely at the *eḍuppu*. This example shows that satisfying all four

constraints, given a single *naḍai* paradigm, is easy to achieve when the number of subdivisions per *tāḷa* cycle ($n_b n_s$) is not equivalent to $0 \ (mod \ 3)$.
13. The *rūpaka tāḷa* has multiple variants. In its shortest form, used here for analysis, it consists of three beats (clap, clap, wave). The second variant is in *viḷambha kāla*, where each beat of the three-beat shorter version is sounded twice to give a total of 6 beats. These two forms of the *tāḷa* are important for performance due to their applicability to many compositions. A third variant of *rūpaka tāḷa* occurs within the *suḷādi sapta tāḷa* and consists of two *angas*—a *dhṛta* (clap + wave) and *laghu* (clap + finger counts). While the *dhṛta* consists of two beats, the *laghu* can consist of either three, four, five, seven, or nine beats depending on its *jāti*. This variant of the *rūpaka tāḷa* is not widely applicable in concert settings. The third variant also has *viḷambha kāla* versions where the number of beats is doubled. For the work presented here, we consider only the first, 3-beat variant of the *tāḷa*.
14. Note that changing the shape of the *pūrvārdha* will not provide a solution to the central problem, as the *pūrvārdha* will still need to be repeated three times along with the rest of the *kōrvai*. Thus, the total number of subdivisions in the composition will still be a multiple of three. Similar arguments can show that the central problem cannot be addressed by trivially choosing a different *uttarārdha* structure.
15. *Tāḷa*s with a large number of beats per cycle can present such a challenge. An extreme example is the *simhanandana tāḷa*, which is the thirty-seventh of the *asṭḥottara ṣata tāḷa*s and contains 128 beats per cycle. In the *caturaṣra naḍai*, it contains 512 subdivisions per cycle, which is not a multiple of three. In the trivial situation where $n_{shift} = 0$, playing a *kōrvai* containing 512 or more subdivisions thrice will be extremely time consuming and, likely, will not be well received by the audience. In such situations, a conventional *kōrvai* structure using inputs from Tables 3 and 4 over multiple *āvartana*s would not work. However, adopting our second new *kōrvai* design with a progressively growing *uttarārdha* (described later in paragraphs [46]–[48] and [54]) makes it possible to play a complete *kōrvai* trio within one cycle of the *tāḷa*. With reference to the descriptions of Design 2, one of the possible solutions is $a_1 = 24, \ x_1 = 12, \ x_2 = 0, \ x_3 = 10$. Hence, the newer approaches to be presented here are also relevant to *tāḷa*s whose total number of subdivisions per cycle is not a multiple of 3.
16. The success and popularity of quantitative musical approaches is unsurprising, given that the source of musical creations is human thought, which has strong algorithmic properties (Cope 2015).
17. In other words, the *pūrvārdha* could minimally consist of three phrases of length a_1, while the *uttarārdha* could minimally include three phrases of length x_1 (where $x_2 = 0$ in the $(x_1, x_2, x_1, x_2, x_1)$ structure). See Tables 3–4.
18. Other Modern and Contemporary Western art music, of course, draws upon non-equally-tempered and/or tonally influenced pitch and harmonic syntaxes, and may explore new timbral possibilities.
19. Composer Igor Stravinsky argued in favor of external compositional constraints more broadly, writing that “the more constraints one imposes, the more one frees one’s self of the chains that shackle the spirit” (Stravinsky [1947] 2003, 65). Straus (2004) suggests that “devising appropriate constraints was, for Stravinsky, an integral part of the compositional or, more properly, precompositional process” (44).
20. In Carnatic music, rhythmic patterns lasting *n* subdivisions typically occur in a limited number of ways, each corresponding to a particular *konnakkol* pattern (Vedavalli 1995; Krishnamurthy 2021). For instance, a 7-subdivision group frequently occurs as the even pattern “tha ka dhi mi tha ki ṭa” or the uneven pattern “tha – dhi – gi na thom”. Moreover, Carnatic *rāga*s include not only specific pitches, but characteristic ornaments (*gamaka*s), phrases (*prayōga*s), and extramusical associations. The more of these rhythmic and melodic parameters a composer implements, the closer a *kōrvai* will be to Carnatic usages.
21. The sixth and seventh notes of the scale, F and G, sometimes occur in their natural form and are sometimes raised to F# and G#, respectively. As such, the melody has a strong melodic minor flavor.